Supplemental Information: Interdependence of Growth, Structure, Size and Resource Consumption During an Economic Growth Cycle

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SI.1 Input-Output Format of Model

Table S.1: Input-Output matrix with value added and net output.

		Goods	Extraction	Net Output: Consumption	Net Output: Investment	Net Output: Change in Value of Inventory	Gross Output
	Goods	$P_g a_{gg} X_g$	$P_g a_{ge} X_e$	C_g	$P_g(I_g^g + I_e^g)$	ΔINV_g	$P_g X_g$
	Extraction	$P_e a_{eg} X_g$	$P_e a_{ee} X_e$	C_e	–	ΔINV_e	$P_e X_e$
Value Added	Profit	Π_g	Π_e				
Value Added	Wages	wL_g	wL_e				
Value Added	Interest Payments	$r_L D_g$	$r_L D_e$				
Value Added	Depreciation	$P_g \delta K_g$	$P_g \delta K_e$				
	Gross Output	$P_g X_g$	$P_e X_e$				

SI.2 Stock and Flow Consistency: Balance Sheet, Transactions, and Flow of Funds Table

The model is stock-flow consistent in both money and physical units of resources and goods. Table S.2 shows the balance sheet, transactions, and flow of funds tables which are the same as in King (2020) [1]. The framework is similar to the Bank, Money, World model of Godley and Lavoie (2007) except we allow firms to have profit [2]. Household deposits, M^h, equal total firm debt. The net worth of firms is the value of their capital minus their debt. We model banks as having zero saving ($S^b = 0$) where bank net interest ($\Pi_b = r_L D - r_M M$) flows to households as bank dividends. Thus, we assume banks have zero net worth ($X^b = 0$).

Table S.2: *Productive sectors*: Balance sheet, transactions, and flow of funds for *productive sectors* as formulated in the macroeconomic model where each of the productive sectors make individual investment decisions and profits.

	Extraction Firms		Goods Firms	
Balance Sheet				
Capital	$P_g K_e$		$P_g K_g$	
Deposits				
Debt (Loans)	$-D_e$		$-D_g$	
Sum (net worth)	X_e^f		X_g^f	
Transactions	Current	Capital	Current	Capital
Consumption	C_e		C_g	
Investment		$-P_g I_g^e$	$P_g I_g^e + P_g I_g^g$	$-P_g I_g^g$
Change in value of inventory	ΔINV_e	$-\Delta INV_e$	ΔINV_g	$-\Delta INV_g$
Intermediate Sales	$+P_e a_{eg} X_g + P_e a_{ee} X_e$		$+P_g a_{ge} X_e + P_g a_{gg} X_g$	
Intermediate Purchases	$-P_g a_{ge} X_e - P_e a_{ee} X_e$		$-P_e a_{eg} X_g - P_g a_{gg} X_g$	
[Value Added]	$[V_e]$		$[V_g]$	
Wages	$-W_e$		$-W_g$	
Depreciation Allowance	$-P_g \delta K_e$	$P_g \delta K_e$	$-P_g \delta K_g$	$P_g \delta K_g$
Interest on debt (loans)	$-r_L D_e$		$-r_L D_g$	
Interest on deposits				
Bank Dividends (net interest)				
Financial Balances	Π_e	$P_g(\delta K_e - I_g^e) - \Delta INV_e$	Π_g	$P_g(\delta K_g - I_g^g) - \Delta INV_g$
Flow of Funds				
Change in capital stock		$P_g \dot{K}_e$		$P_g \dot{K}_g$
Gross Fixed Capital Formation		$P_g I_g^e$		$P_g I_g^g$
Change in deposits				
Change in debt (loans)		$-\dot{D}_e$		$-\dot{D}_g$
Column sum		Π_e		Π_g
Change in net worth		$\dot{X}_e^f = \Pi_e$		$\dot{X}_g^f = \Pi_g$

Table S.2: (continued) *Households, Banks, and Sum*: Balance sheet, transactions, and flow of funds tables.

	Households	Banks	Sum
Balance Sheet			
Capital			$P_g(K_e + K_g)$
Deposits	M^h	$-M$	
Debt (Loans)		D	
Sum (net worth)	X^h	$X^b = 0$	$X^{tot} = P_g(K_e + K_g)$
Transactions			
Consumption	$-C$		
Investment			
Change in value of inventory			
Intermediate Sales			$+P_e a_{eg} X_g + P_e a_{ee} X_e + P_g a_{ge} X_e + P_g a_{gg} X_g$
Intermediate Purchases			$-P_g a_{ge} X_e - P_e a_{ee} X_e - P_e a_{eg} X_g - P_g a_{gg} X_g$
[Value Added]			
Wages	W		
Depreciation Allowance			
Interest on debt (loans)			
Interest on deposits	$r_M M^h$	$-r_M M$	
Bank Dividends (net interest)	Π_b	$-\Pi_b$	
Financial Balances	S^h	$S^b = 0$	0
Flow of Funds			
Change in capital stock			
Gross Fixed Capital Formation			
Change in deposits	$\dot{M}^h$	$\dot{M}$	
Change in debt (loans)		$\dot{D}$	
Column sum	S^h	S^b 8	
Change in net worth	$\dot{X}^h = S^h$	$\dot{X}^b = S^b = 0$	

SI.3 Additional Equations not in Methods

SI.3.1 Consumer Price Index (CPI) and GDP deflator

We calculate inflation as a weighted change in the price of each sector output consumed by households using Equation 15. The consumer price index (CPI) is as in Equation S.1.

$$CPI = \prod_{t=1}^{T} (1 + i_t) \tag{S.1}$$

Assume defining the GDP deflator as nominal GDP divided by real GDP (where $Y_{i,t}$ is the net monetary output of sector i at time t). $P_{i,o}$ is the initial price of output from sector i. Real net output, value added, investment, debt, and consumption are calculated by dividing nominal values by the GDP deflator (e.g., $Y_{g,real} = Y_{g,t}/\text{GDP deflator}_t$).

$$\begin{aligned}\text{GDP deflator}_t &= \frac{P_{g,t}\left(\frac{Y_{g,t}}{P_{g,t}}\right) + P_{e,t}\left(\frac{Y_{e,t}}{P_{e,t}}\right)}{P_{g,o}\left(\frac{Y_{g,t}}{P_{g,t}}\right) + P_{e,o}\left(\frac{Y_{e,t}}{P_{e,t}}\right)} \\ &= \frac{Y_{g,t} + Y_{e,t}}{P_{g,o}\left(\frac{Y_{g,t}}{P_{g,t}}\right) + P_{e,o}\left(\frac{Y_{e,t}}{P_{e,t}}\right)} \end{aligned} \tag{S.2}$$

SI.3.2 Death Rate

Equation S.3 describes the function for death rates, where s is the per capita resource consumption threshold below which death rates rise. As physical per capita resource consumption, $\frac{C_e}{NP_e}$, declines below the threshold to zero the death rate linearly, the death rate increases from a minimum value of α_m to a maximum "famine" death rate of α_M as in [3].

$$\alpha_N\left(\frac{C_e}{NP_e}\right) = \alpha_m + \max\left(0, 1 - \frac{\left(\frac{C_e}{NP_e}\right)}{s}\right)(\alpha_M - \alpha_m) \tag{S.3}$$

SI.3.3 Inventory Coverage and Capacity Utilization

Perceived inventory coverage for each sector:

$$\begin{aligned} IC_{e,perceived} &= \frac{\frac{\text{perceived wealth, } w_H}{\text{time delay}}}{\text{targeted consumption of resources}} \\ IC_{e,perceived} &= \frac{\frac{w_H}{\tau_{IC,e}}}{C_e/P_e + a_{eg}X_g + a_{ee}X_e} \end{aligned} \tag{S.4}$$

$$\begin{aligned} IC_{g,perceived} &= \frac{\frac{\text{perceived goods, } g}{\text{time delay}}}{\text{targeted consumption of goods}} \\ IC_{g,perceived} &= \frac{\frac{g}{\tau_{IC,g}}}{(C_g + I_e + I_g)/P_g + a_{gg}X_g + a_{ge}X_e} \end{aligned} \tag{S.5}$$

Perceived capacity utilization of each sector, $0 \leq CU_{i,perceived} \leq 1$, is a lookup table that is an increasing function of the inverse of its respective inventory coverage [4]. When there is more inventory, capacity utilization decreases, and vice versa. The reference inventory coverage, $IC_{ref,i}$, is defined in the lookup table for capacity utilization as the amount of inventory present for capacity utilization to be at its reference value, $CU_{ref,i}$. We set $CU_{ref,i} = 0.85$ at $IC_{ref,i} = 1$.

$$CU_{i,perceived} = f(IC^{-1}_{i,perceived}) \tag{S.6}$$

The lookup table for CU of both sectors use input values as $IC^{-1}_{i,perceived} = [0, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 1e6]$ and output values as $CU_{i,perceived} = [0, 0.30, 0.55, 0.75, 0.85, 0.90, 0.94, 0.98, 0.99, 1, 1]$.

SI.3.4 Phillips Curve (Wage Function)

For $\phi(\lambda_N)$ we use Keen's nonlinear exponential curve (Equation S.7) that allows wages to rise increasingly rapidly at high participation but decrease slowly at low participation rate [5]. In Equation S.7 $\phi_{min} < 0$ is the minimum decline in wages at low participation rate, ϕ_o is the change in current wage defined at $\lambda_{N,o}$ (typically set $\phi_o = 0$ at an equilibrium participation rate $\lambda_{N,o}$), and ϕ_s defines the exponential rate of increase.

$$\phi(\lambda_N) = (\phi_o - \phi_{min}) e^{\frac{\phi_s}{(\phi_o - \phi_{min})}(\lambda_N - \lambda_{N,o})} + \phi_{min} \tag{S.7}$$

SI.3.5 Solving for Lagged Variables

For each sector i, the variables modeled using a first order lag are the capacity utilization (CU_i), perceived inventory coverage ($IC_{i,perceived}$), price (P_i), net output (Y_i), profit (Π_i), value added (V_i), and value of inventory (INV_i). Thus, each of $CU_i, IC_{i,perceived}, P_i, Y_i, \Pi_i, V_i$, and INV_i is modeled as an extra model state. In the model code, these lagged states are used to inform investment and all inputs needed to solve for a new "current" price. Additional model calculations use this newly calculated price, including calculations that updated the lagged states themselves. The states are updated via Equation S.8. For the inventory coverage of each sector (IC_i) it is the perceived inventory coverage ($IC_{i,percieved}$) that is modeled as a lagged state on the left hand side.

$$\dot{n}_{lagged} = (n - n_{lagged})/\tau \tag{S.8}$$

SI.3.6 Net Power Accounting

Considering the flow of extracted and consumed resources as a flow of "power" (energy per time), we summarize the power return ratios (PRR) used in King (2020) [1, 6]. PRRs characterize the power generated by the energy (i.e., resources) sectors relative to the power consumed by the energy sectors themselves. This self-consumption can include both operating inputs and energy embodied in investment. Researchers have speculated on minimum levels of PRRs "required" to sustain society [7], and calculation of PRRs within our model allows internally-consistent investigation of their relation to economic growth, structure, and population. The net external power ratio (NEPR) is defined in Equation S.10 as net resource extraction divided by the extraction sectors' own use of resources.

NEPR represents what is often termed "energy return on energy invested" (EROI) in much of the literature [6]. Since our PRR calculation uses instantaneous resource flow rates in the numerator and denominator, the term *power* return ratio is more appropriate than energy return ratio (ERR) (e.g., energy is power integrated over time). However, much of the net energy literature uses the terminology EROI and ERR to refer both to ratios of energy and ratios of power.

We calculate the resources embodied in extraction capital, K_e, via a resource intensity, ϵ, that measures gross resources extraction required per net physical output of each sector (see [8], [9], and [10] for the methodology for calculating "energy intensities"). In Equation S.9, $\hat{y}_{\text{extract}}$ is a 2×2 diagonal matrix with non-zero elements *only* for the gross extraction by sectors that extract resources from the environment. For our model $\hat{y}_{\text{extract},ee} = \hat{y}_{\text{extract},22} = X_e$, $\hat{y}_{\text{extract},gg} = 0$, $\hat{X}$ is a diagonal matrix of the gross physical output of each sector (X_g and X_e), $\mathbf{1}$ is the identity matrix, $\mathbf{A}$ the technical coefficients matrix, and matrix E is a 2×2 matrix of resource intensities, ϵ_{ij}. Further, the first row of E is zero, and the second row contains the resource intensities of ϵ_{eg} representing the gross resource input per unit of net physical goods output, $\frac{Y_g}{P_g}$, and ϵ_{ee} representing the gross resource input per unit of net physical resources output, $\frac{Y_e}{P_e}$. Thus, the embodied resources in extraction capital each time step is equal to $\epsilon_{eg}\frac{I_e}{P_g}$. Recall that $X_e = \delta_y y K_e CU_e$ as well as that $\frac{I_e}{P_g}$ has units of goods and represents the physical goods allocated to become new extraction capital.

$$\begin{bmatrix} 0 & 0 \\ \epsilon_{eg} & \epsilon_{ee} \end{bmatrix} = E = \hat{y}_{\text{extract}} \hat{X}^{-1} (\mathbf{1} - A)^{-1} \tag{S.9}$$

$$\begin{aligned} NEPR &= \frac{\text{resource extraction} - \text{resources required to invest in } K_e - \text{resources required to operate } K_e}{\text{resources required to invest in } K_e + \text{resources required to operate } K_e} \\ &= \frac{X_e - \epsilon_{eg}\frac{I_e}{P_g} - a_{ee}X_e}{\epsilon_{eg}\frac{I_e}{P_g} + a_{ee}X_e} \end{aligned} \tag{S.10}$$

Per King *et al.* (2015) we can consider an economy level net power ratio ($\text{NPR}_{\text{economy}}$) as similar to but distinct from the sector-specific NEPR [6]. With only one extraction ("energy") sector, $\text{NPR}_{\text{economy}}$ is calculated using only ϵ_{ee} in Equation S.11. Economy-wide gross power ratio ($\text{GPR}_{\text{economy}}$) equals one plus $\text{NPR}_{\text{economy}}$ [11], and it has been referred to as the EROI of the economy [12]. The upper limit for $\text{NPR}_{\text{economy}}$ is defined using a_{ee} as $\text{NPR}_{\text{economy, uppper limit}} = \frac{1-a_{ee}}{a_{ee}}$, where $1 - a_{ee}$ is the fraction of extracted resources left for all other economic activity after operating extraction capital [8]. As such, $\text{NPR}_{\text{economy, uppper limit}}$ is largely defined by the resource efficiency of extraction sector capital.

$$NPR_{\text{economy}} = \frac{1}{\frac{\text{gross power extracted}}{\text{net power output}} - 1} = \frac{1}{\epsilon_{ee} - 1} \tag{S.11}$$

SI.3.7 Decreasing Resource Consumption to Operate Capital (η_i) as Function of Cumulative Investment

In some simulations we explore the concept an increase in resource consumption efficiency of capital. We do this by decreasing η_i as a function of physical capital investment in sector i as in Equation S.12. The equation approximates a logistic decrease in η_i as a function of cumulative capital investment, $I_{i,cumulative}$ (Equation S.14).

When simulating, we choose a time "$T_{critical}$" at which to start the process of decreasing η_i and calculating $I_{i,cumulative}$. In the paper this critical time is $T = 0.1$, or the first time step in the simulation. Before this time, we assume $I_{i,cumulative} = 0$ such that there is no investment that yet contributes to "learning by doing" that would decrease η_i. After this time, $I_{i,cumulative}$ is the integral of gross physical capital investment (without depreciation).

The parameter $\eta_{i,adder}$ (Equation S.13) adjusts adjusts the baseline level. Parameter S adjusts the steepness of the curve. $I_{i,mid}$ sets the approximate mid-point, or inflection point, of the logistic curve. The maximum and minimum values of η_i are $\eta_{i,max}$ and $\eta_{i,min}$, respectively, where the starting value is by definition $\eta_{i,max}$.

$$\eta_i = \eta_{i,adder} + \frac{\eta_{i,min} - \eta_{i,max}}{1 + e^{S(I_{i,cumulative} - 2I_{i,mid})}} + \eta_{i,max} \tag{S.12}$$

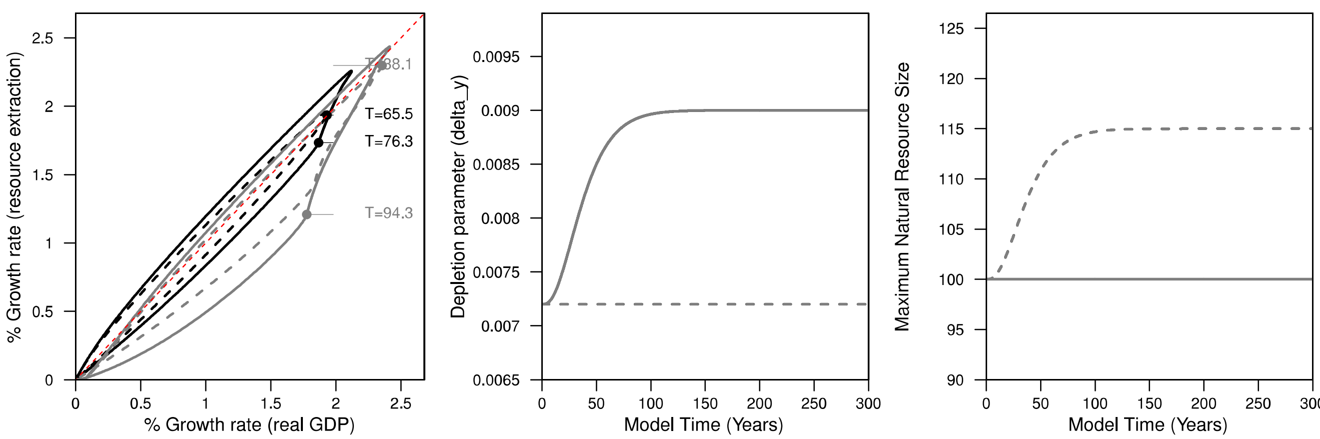
$$\eta_{i,adder} = \eta_{i,max} - \left(\frac{\eta_{i,min} - \eta_{i,max}}{1 + e^{S(I_{i,cumulative} - 2I_{i,mid})}} + \eta_{i,max} \right) \tag{S.13}$$

$$\begin{aligned} \dot{I}_{i,cumulative} &= 0, \; (T < T_{critical}) \\ \dot{I}_{i,cumulative} &= \frac{(1 - fP_{I_i})I_i}{P_g}, \; (T \geq T_{critical}) \end{aligned} \tag{S.14}$$

SI.4 Additional Simulation Results

SI.4.1 Comparing of results when increasing δ_y versus increasing λ_y

Figure S.1 shows that by only looking at data for growth rates for resource (or energy) consumption and GDP, it is difficult to impossible to explain the difference between growth induced via increased technological capability (increasing δ_y in scenarios FC-000 and FC-100, Figure S.1(b)) to access a constant maximum resource size versus the ability to access a larger resource at constant technology (increasing $\lambda_{y,max}$ in scenarios "FC-000 alternative" and "FC-100 alternative", Figure S.1(c)). The general pattern in the growth rates for resource extraction and GDP follow the same counter-clockwise pattern and change from superlinear (above 1:1 line) to sublinear (below 1:1 line) scaling. Real world data most assuredly exhibit influence from both drivers of accessing more resources and the ability to access known resources more fully.



(a) (b) (c)

Figure S.1: (a) HARMONEY simulation results for FC-000 (black solid), similar to "FC-000 alternative" (black dashed), FC-100 (gray solid), and "FC-100 alternative" (gray dashed) scenarios. (b) FC-000 and FC-100 spur growth by increasing δ_y from 0.0072 to 0.009. (c) The "alternative" scenarios spur growth by increasing the maximum resource size $\lambda_{y,max}$ from 100 to 115.

SI.4.2 Variations on Full Cost Scenarios with Lower Wage Bargaining and Ponzi

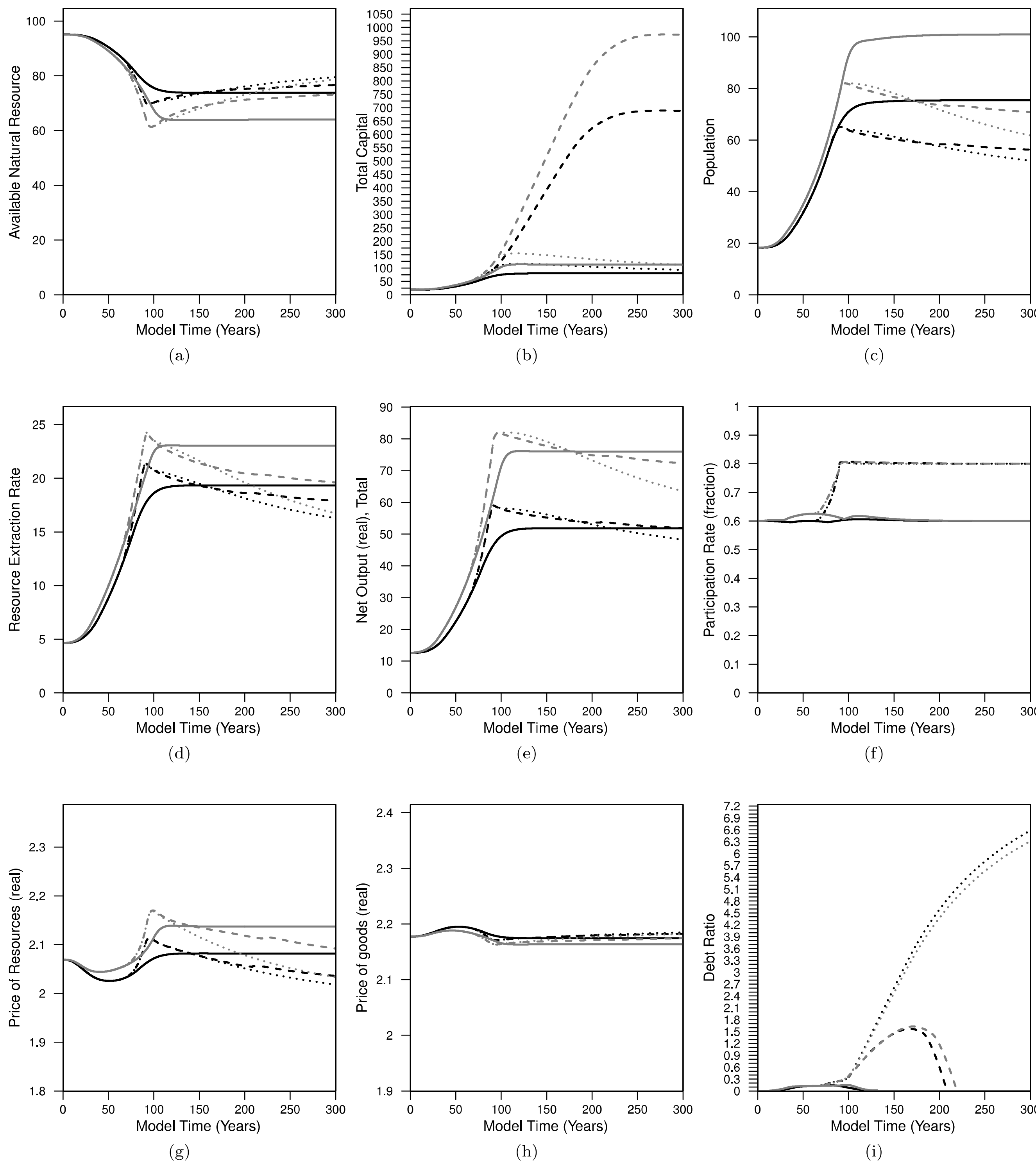


Figure S.2: Scenarios FC-000, FC-010, FC-011, FC-100, FC-110, FC-111 (black, black dashed, black dotted, gray, gray dashed, gray dotted) results when setting prices via a constant markup ($\mu_i = 0.13$) with $\kappa_0 = 1.0$ and $\kappa_1 = 1.5$. (a) available resources (in the environment), (b) total capital, (c) population, (d) resource extraction rate, (e) total real net output, (f) participation rate, (g) real price of extracted resources, (h) real price of goods, (i) debt ratio, and (continued) ...

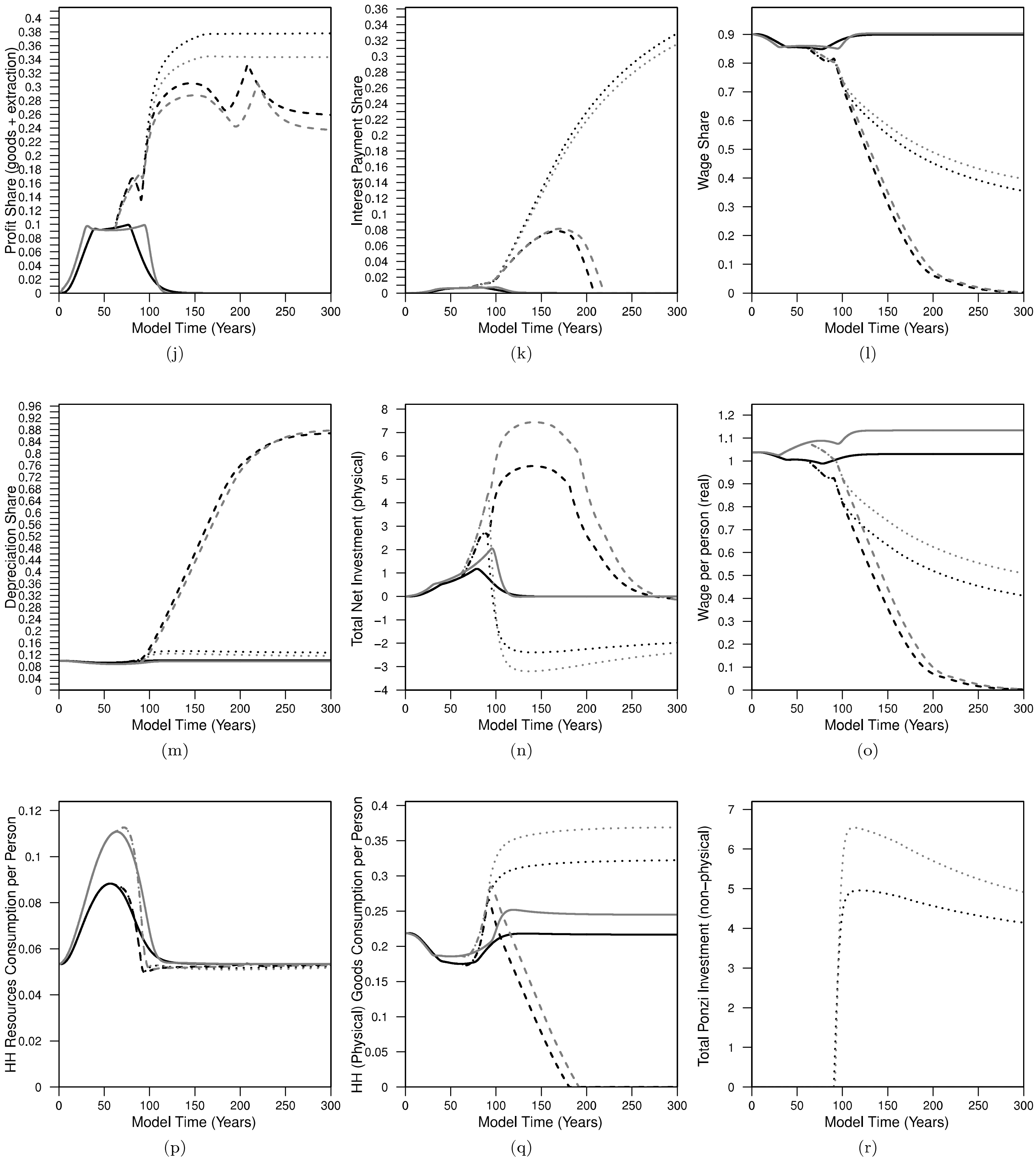


Figure S.2: (continued) (j) profit share, (k) interest share, (l) wage share, (m) depreciation share, (n) physical net investment in new capital, (o) real wage per person, (p) household consumption of (physical) resources per person, (q) household consumption of (physical) goods per person, (r) total non-physical Ponzi investment,

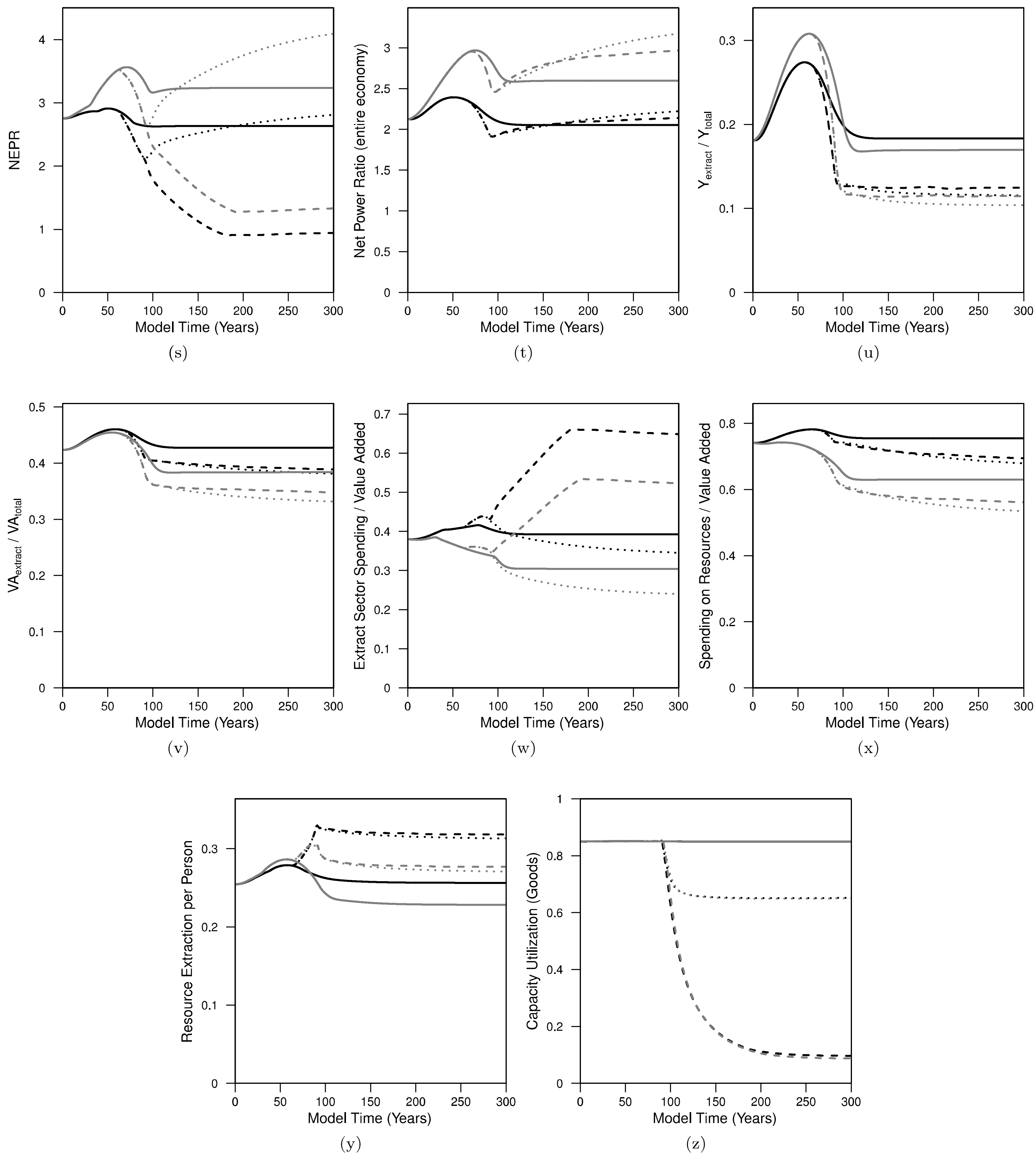


Figure S.2: (continued) (s) net external power ratio (extraction sector, NEPR), (t) net power ratio (entire economy, NPR), (u) fraction of net output from extraction sector, (v) fraction of value added in extraction sector, (w) extraction sector spending per total value added (= per total net output), (x) spending on resources per total value added (= per total net output), (y) total resources extraction per person, and (z) capacity utilization of goods capital.

SI.4.3 Information Theory Calculations for Full Cost Scenarios

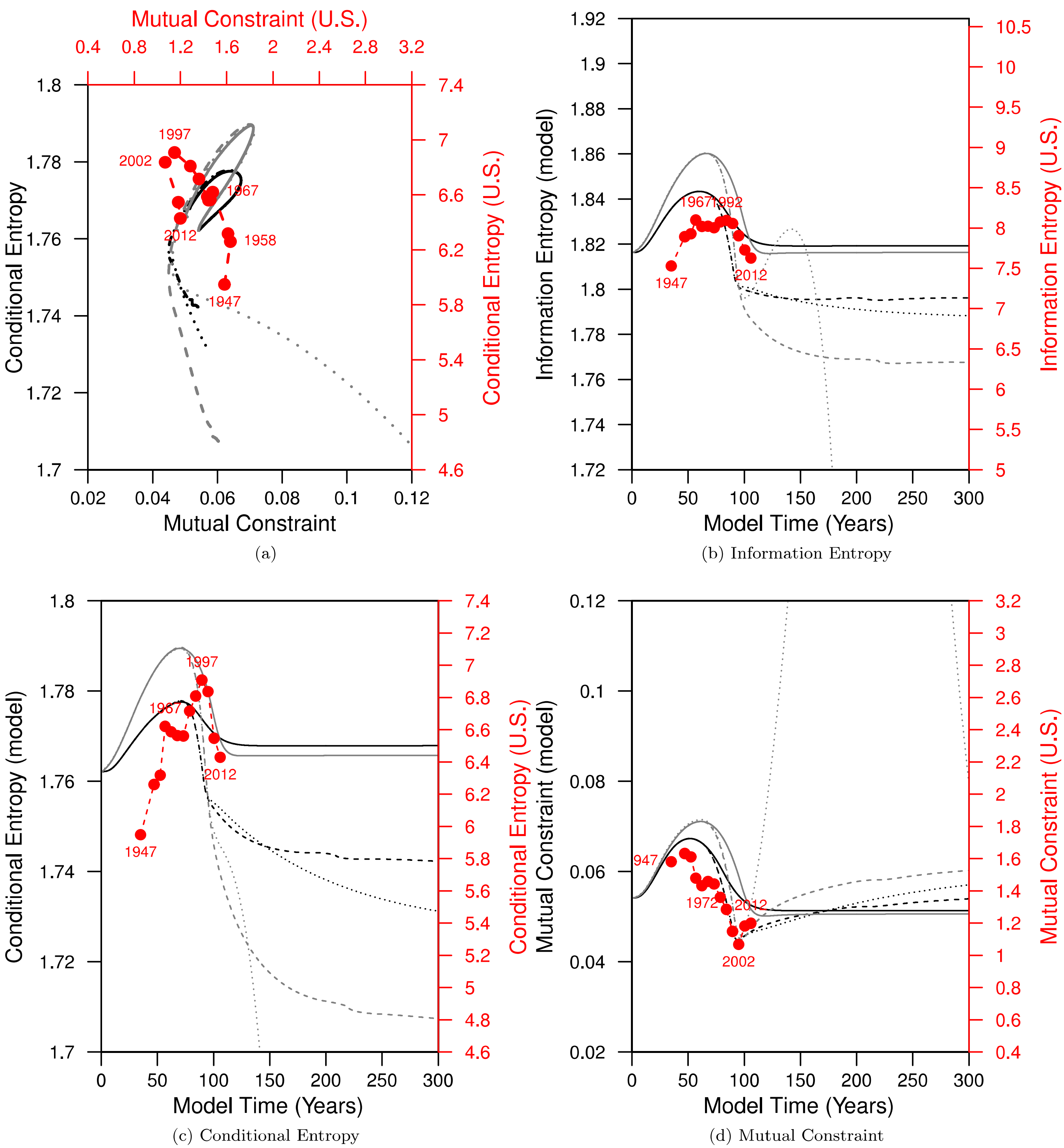
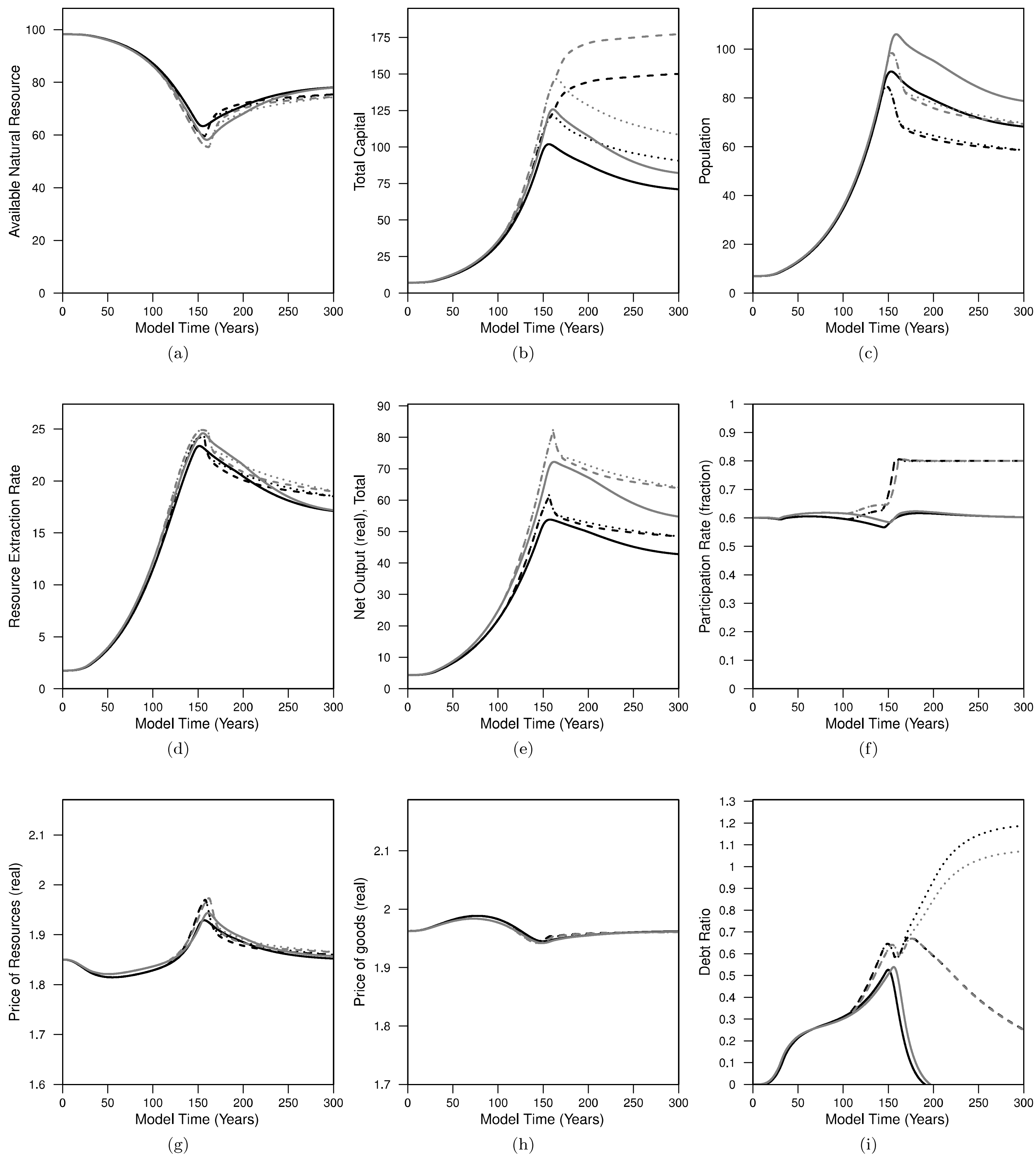


Figure S.3: Information theoretic metrics of the full cost scenarios compared to the 37-sector aggregation of the U.S. Use tables from 1947–2012 from King (2016) (red dashed lines, right axis) [13]. FC-000 (black solid), FC-010 (black dashed), FC-011 (black dotted), FC-100 (gray solid), FC-110 (gray dashed), FC-111 (gray dotted). (a) Conditional entropy versus mutual constraint, (b) information entropy vs time, (c) conditional entropy vs time, and (d) mutual constraint vs time. U.S. information theory metrics are calculated using base 2 logarithm instead of natural logarithm as in [13].

SI.4.4 Variations on Marginal Cost Pricing Scenarios with Lower Wage Bargaining and Ponzi



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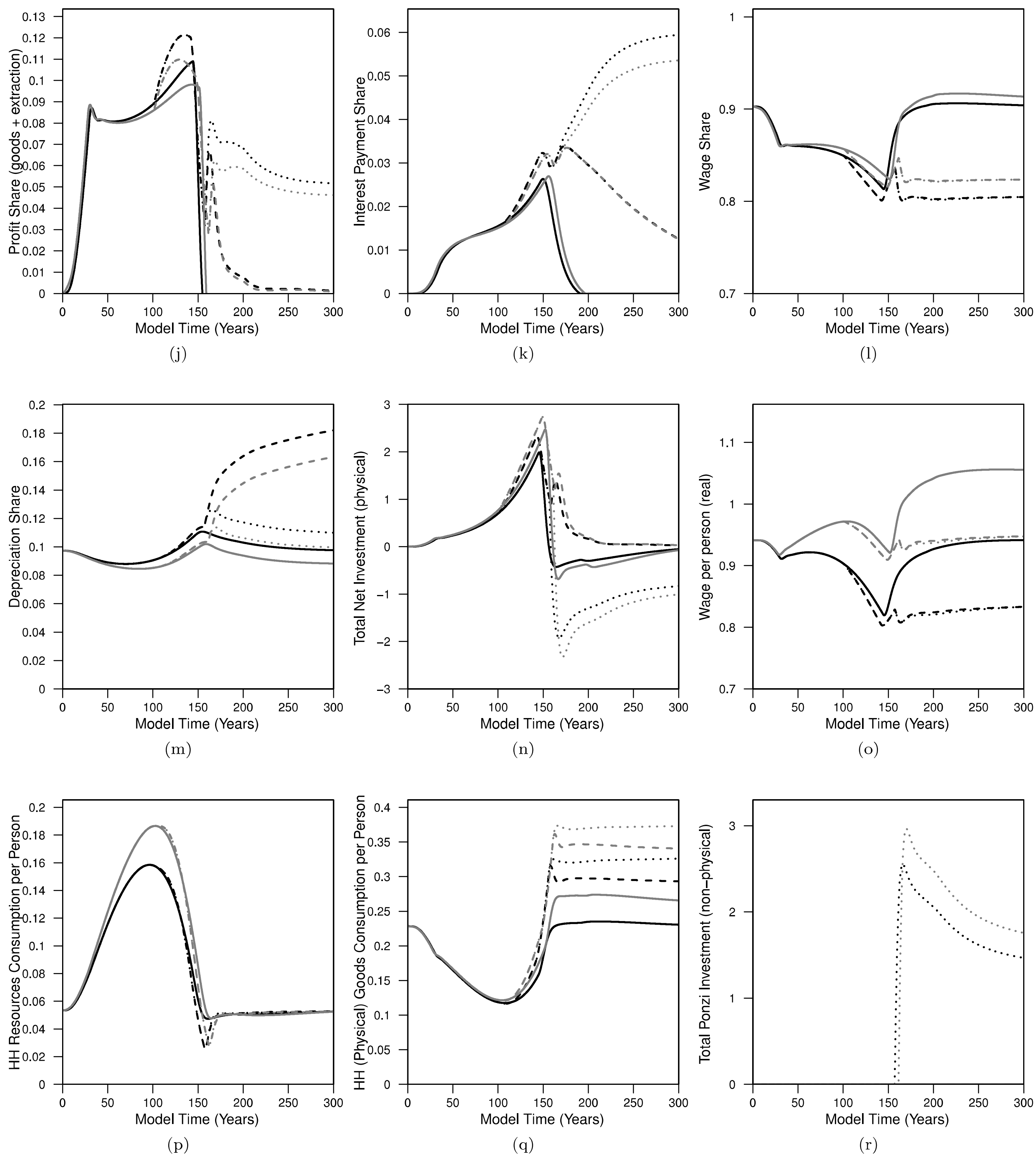

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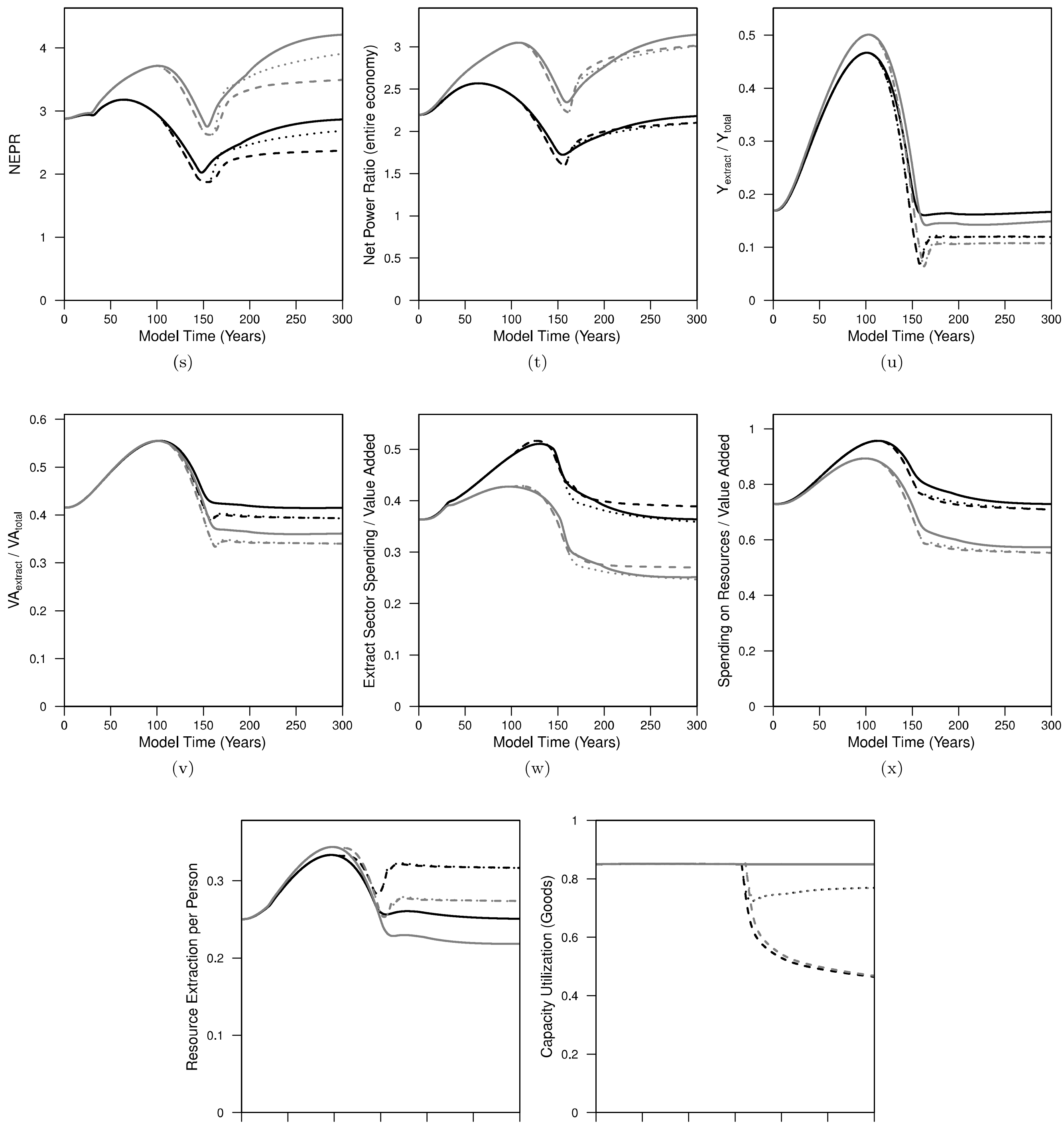

Figure S.4: (continued) (s) net external power ratio (extraction sector, NEPR), (t) net power ratio (entire economy, NPR), (u) fraction of net output from extraction sector, (v) fraction of value added in extraction sector, (w) extraction sector spending per total value added (= per total net output), (x) spending on resources per total value added (= per total net output), (y) total resources extraction per person, and (z) capacity utilization of goods capital.

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