An Integrated Biophysical and Economic Modeling Framework for Long-Term Sustainability Analysis: The HARMONEY Model

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Abstract

This paper derives a long-term dynamic growth model that endogenously links biophysical and economic variables in a stock-flow consistent manner. The two industrial sector model enables exploration of interdependencies among resource extraction rate and depletion; the accumulation of population, capital, and debt; and the distribution of money flows within the economy. Using a post-Keynesian economic framework, we find that wage share declines after the model reaches a constant per capita resource extraction rate, with the level of investment and markup on costs determining the rate of decline. This pattern is consistent with data for the United States. Thus, the model framework enables realistic investigation of trade-offs between economic distribution, size, and resources consumption between sectors as well as between labor and capital. These trade-offs are core to the debates regarding environmental and socioeconomic sustainability. This model serves as a platform upon which to add features to explore long-term sustainability questions such as a transition to low-carbon energy.

Keywords: energy, resources, macroeconomics, post-Keynesian, dynamics, long-term growth

1 Introduction

Biophysical models describe trends in energy and natural resource use, population cycles and growth, and environmental sustainability related to resource use or pollution. They simulate natural system dynamics but often ignore major economic processes and monetary flows [1, 2, 3]. Economic models describe outcomes such as economic growth rates, employment, capital and debt accumulation, and wages but often neglect or exogenously assume quantities and constraints related to natural resource extraction and population. Because biophysical and economic systems influence each other through interlinking feedbacks, the two approaches should be combined [**?**]. This paper describes a model that performs this combination in a novel way.

The novelty of the model lies not in the conception of any individual component, but in the design and integration of the components. The goal for the present model is to provide a common framework that enables simulation of long-term (multi-decadal) macrodynamics while endogenously including critical macroeconomic and biophysical factors. To acheive this goal we specifically avoid neoclassical growth theory and use a post-Keynesian model formulation with stock and flow consistency for both monetary accounts and a physical natural resource [4]. The model is highly aggregated, conceptual, and not calibrated to a particular economy. The model treats economic growth, size, and structure as interdependent. With this model framework we wish to better enable economists and applied and natural scientists to communicate on policy-relevant issues relating to natural resources use, growth of economic output and debt, economic distribution, and population growth. To that end the present model serves as a base from which to add features and calibrate to real-world economies to explore future energy-economic scenarios, such as a low-carbon energy transition.

The motivation for a new biophysical macreconomic model is to provide a viable option that overcomes longstanding and well-documented objections to modeling economic growth using the neoclassical theoretical framework. The majority of macroeconomic growth models use the Cobb-Douglas aggregate production function, often in the form of the Solow-Swan growth model [5]. Energy and natural resource inputs, while not precluded from the neoclassical framework, are ignored within the Solow-Swan production function from which total factor productivity (TFP) is derived as the "technology" residual after accounting for growth contributions from only capital and labor. Felipe and Fisher demonstrate that there is no theoretical foundation for this exogenous technology Cobb-Douglas growth equation, and that TFP is a mathematical identity rather than an independent measure of technological change [6, 7, 8]. One can directly incorporate energy as a third factor of production into a Cobb-Douglas or more general aggregate production functions, such as the constant elasticity of substitution function. The latter introduces ambiguities in how to nest more than two factors [9]. Nonetheless, Brada (2013) notes how the theoretical structure of neoclassical aggregate production functions largely precludes resource exhaustion as a potentially significant causal explanation for a slowdown in economic growth, labor productivity, or TFP [10].

To emphasize the policy-level importance of realistically modeling energy use within long-term growth models, we briefly discuss a critical problem with using neoclassical theory for modeling of a low-carbon energy transition. Dozens of Integrated Assessment Models (IAMs) inform climate change policy, including those used within Working Group III of the Intergovernmental Panel on Climate Change (IPCC) [11, 12]. IAMs integrate models of the economy to models of global climate. However, practically all IAMs use neoclassical growth theory and thus share the critical weaknesses previously stated [13]. Most important, productivity is usually specified exogenously through autonomous contributions to total factor productivity (TFP), or some other productivity factor. Many IAMs assume TFP grows between one and two percent per year. This assumption dominates economic growth such that no matter the magnitude and rate of a transition to low-carbon energy, economic growth is not only reported as positive, but robust. For example, the IAM outputs shown in IPCC Assessment Report 5 imply "... that reaching about 450 ppm CO_2eq by 2100 would entail global consumption losses of ... 3% to 11% in 2100 ..." against baseline consumption in 2100 of 300% to 900% relative to 2010 [12]. To limit atmospheric greenhouse gas concentrations to 450 ppm CO_2eq by 2100, climate models suggest the entire global economy needs to have annual emissions near or even below zero by that year. Thus, the report states that a world with zero or negative total carbon emissions in 2100 will have an economy only trivially smaller than if no climate change or carbon transition occurred. As critics have indicated, this assumes significantly different energy-economic patterns, making the assertion highly suspect [14]. A major reason and problem is that by its construction, TFP and economic growth are independent of changes in the energy system itself (Figure A.II.1 of [15]). Instead of assuming changes to the energy system and calculating effects on economic factors, most IAMs first assume economic growth and fill in options for the energy system. This is not the type of modeling we need.

The present model is informed by data and research emphasizing that TFP and GDP growth are highly coupled to energy consumption. Global data for primary energy and gross world product (GWP) since 1965 show that the output elasticity of primary energy is near 0.7 (i.e., a 10% change in primary energy is associated with a 7% change in GWP). Neoclassical theory states this elasticity should be equal to the cost share of energy, which is typically below 0.1. Clearly the output elasticity of primary energy is not equal to its cost share, and we should seek theory that does not assume the two are equal. By removing the neoclassical constraint that the output elasticity of each factor input equal its cost share, studies by Ayres, Warr, and Kümmel explain past economic growth by incorporating energy (or exergy[1]) and/or its efficiency of use into aggregate production

[1]Exergy is the portion of energy that has the potential to do physical, or thermodynamic, work relative to a reference environment condition often assumed as standard temperature and pressure. It is the maximum useful work possible from an energy conversion process that brings the system into equilibrium with its environment, or its heat reservoir. Exergy has the same units of energy. Unlike energy, exergy is not conserved.

functions [16, 17, 18, 19, 20]. Ayres has shown that "useful work," equal to end-use energy times its efficiency of use, correlates strongly with gross domestic product in the U.S. Also, the efficiency of converting end-use energy into useful work correlates strongly with TFP estimates, thus reducing the need to explain past economic growth through exogenous contributions to TFP. In addition, the MARCO-UK model of Sakai *et al.* (2019), using a set of econometric equations within a post-Keynesian framework, demonstrates that economic growth in the UK has been significantly affected by gains in the efficiency of converting final energy to useful work [21]. Režný and Bureš (2018) and Court *et al.* (2018) show additional recent efforts that incorporate energy into a neoclassical growth model using some form of physical energetic constraint [22, 23]. Specifically, Court *et al.* (2018) include endogenous technical change in the form of increasing efficiency for the conversion of primary energy into useful work. Santos *et al.* (2018) employ a conventional aggregate production function with capital and labor inputs, but based on the useful work concept of Ayres, they add an energy-related constraint to ensure that operating capital entails consumption of exergy at some efficiency [24].

Clearly much modeling effort concentrates on using aggregate production functions. However, there is value to exploring alternative approaches. Some models have sought this goal. Taylor *et al.* (2016) construct a post-Keynesian climate-economy model in which labor productivity is tied to energy inputs [25]. Berg *et al.* (2015) also create a generalizable post-Keynesian model with two-sectors they specify to produce energy and a generic consumer/industry good [26]. Building on Taylor *et al.* (2016) we construct a post-Keynesian model, and as in Berg *et al.* (2015) we model two sectors, a resource extraction sector and a goods sector that produces a generic capital and consumer good. Building on Ayres' work, Keen *et al.* (2019), Santos *et al.* (2018), and Court *et al.* (2018) we tie energy consumption to the operation of capital, but unlike them we do not model the economy with a single sector aggregate production function. In this paper we model output from each of two sectors via a Leontief production function where physical capital requires resource consumption for operation. Similar to Court *et al.* (2018), we make explicit comparison of economic outcomes to the concept of net energy to better integrate that concept within economic theory. This comparison provides a method to discuss the energetic capabilities of the energy sector, including productivity that is a function of depletion, to the size, growth, and structure of the overall economy.

Many biophysical and economic studies posit the impacts of transitioning from a period of increasing resource consumption to one of steady or decreasing consumption. To that end, our model explicitly incorporates "limits to growth" similar to the World3 model used in the 1972 *Limits to Growth* [3]. A transition from increasing consumption might be disruptive, raising the possibility of financial instability. Thus, it is critical to model debt accumulation such that we can relate resource and financial flows. We do this by following the model proposed by Keen (1995, 2013), and inspired by the Minsky Financial Instability Hypothesis, to explain debt-induced financial instability [27, 28, **?**, 29, 30]. In particular, we follow much of the Bovari *et al.* (2018) economic structure that follows Keen's approach.

Whether any "limits to growth" exist is contested in the economic literature [31, 32], but there is little doubt in the ecological literature [33]. As noted by Martin Weitzman's comments within William Nordhaus' 1992 critique of World3, "There may be a some value in trying to understand a little better why the advocates of the limits-to-growth view see things so differently and what, if anything, might narrow the differences . . . The average ecologist sees everywhere that carrying capacity is a genuine limit to growth. Every empirical study, formal or informal, confirms this truth. And every meaningful theoretical model has this structure built in . . . Needless to say, the average contemporary economist does not readily see any long-term carrying capacity constraints for human beings. The historical record is full of past hurdles to growth that were overcome by substitution and technological progress." In this contest between disciplinary extrapolations into a counterfactual future, it is worth noting that recent reassessments of the original *Limits to Growth* study [34, 3] show the model effectively describes global macro trends that have taken place in the subsequent 40+ years [35, 36]. Bardi (2011) discusses this at length while countering that Nordhaus' 1992 critique was ignorant of the mathematical and computational methods used in World3 [33]. Further, the concepts of "planetary boundaries" [37, 38] and the "Anthropocene" [39] show that it is possible to exceed limits well before they have a significant impact.

Per Weitzman, our model seeks to "narrow the differences" between economic and ecological worldviews. We do this by explicitly including several major economic variables, using stock-flow consistent accounting of monetary flows, into a framework consistent with biophysical flows. However, since the present paper is meant to establish a model structure rather than study a transition from from one technology to another (e.g., fossil to renewable energy), the model includes only one natural resource. This resource is a regenerative stock, akin to a forest. Thus, the present model limits the maximum flow of resource extraction but not the cumulative quantity extracted over all time. Future modifications can include a nonrenewable resource stock as well as resources representative of wind and solar power.

2 Methods

Conceptually, the present model seeks to intergrate the "simplest" biophysical and economic models that include a subset of variables we deem critical for undertanding long-term resource-economic interactions in a modern economy. This subset of variables are *population* and *capital* that require *natural resources* for growth and maintainance, *debt*, *wages*, and *employment*. The core biophysical aspects come from the Human and Nature Dynamics (HANDY) model of Motesharrei *et al.* (2014) [1]. The core economic structure comes from the endogenous money economic Goodwin-Keen model of Steve Keen [27, 28]. Our model can be seen as an integration of these two models, and we call our present model the "HARMONEY" model: Human And Resources with MONEY. The biophysical HANDY model is a Lotka-Volterra predator-prey model in which humans are predators that consume a regenerative resource. The Goodwin-Keen economic model adds an endogenous money supply, as private debt, to Goodwin's model of the business cycle [27, 40]. Like HANDY, Goodwin's model is also of a Lotka-Volterra structure with the wage share (fraction of output as wages) and employment as predator and prey, respectively.

To model how resources flow in an economy, it is crucial that we model at least two sectors, one of which is an extraction (subscript e) sector. The second is a generic consumer/capital goods manufacturing (subscript g). Crucially, both our model and that in Berg *et al.* (2015) requires that each sector must take inputs from both sectors (itself and the other sector) to produce its output. Significant differences from the Berg *et al.* (2015) two-sector model are that we endogenously model population and define a finite size natural resource that, as it depletes, increasingly requires its own output as an operating input. This approach enables the exploration of relationships between the net power of the energy system and economic outcomes (growth, size, and distribution). This linkage between the net power (or energy) of economies and societies is often discussed in papers, but there is little to no theoretical basis for claims of a relationship. This model framework provides the theoretical basis. Uehara *et al.* (2015) build a similar 2-sector model, but unlike us they assume a utility-maximizing representative agent for price formation, do not model private debt, and like HANDY assume extraction is a function of labor only [41]. Unlike in Uehara *et al.* (2015), but as in Berg *et al.* (2015), we model the unit price of output of each sector as a markup (increase) on the unit cost of production.

Figure 1 illustrates the major flows of natural resources, goods, and money that relate to capital investment, capital operation, and household consumption. For clarity, not all model stocks and flows are explicitly indicated in the figure. The neglected flows are the intermediate goods input flows to operate each type of capital and the flows of population (i.e., births and deaths). Also not shown are stocks of inventories, which act as buffers in the model. The dashed gray box indicates stocks and flows that are common to the HANDY model of Motesharrei *et al.* (2014).

2.1 HARMONEY Model Description

2.1.1 Production

Each sector i operates a stock of physical capital, K_i, that produces a gross output of physical product, X_i, and requires a quantity of natural resource, η_i, for operating capital at full capacity utilization, CU_i. Gross output from each sector i assumes a Leontief production function (Equations 1 and 2), where L_i and a_i are the labor and labor productivity, respectively. Output can be constrained directly by total economy-wide labor, L, or the capital of each sector, and if labor constrains, then capacity utilization decreases to equate the two terms in each

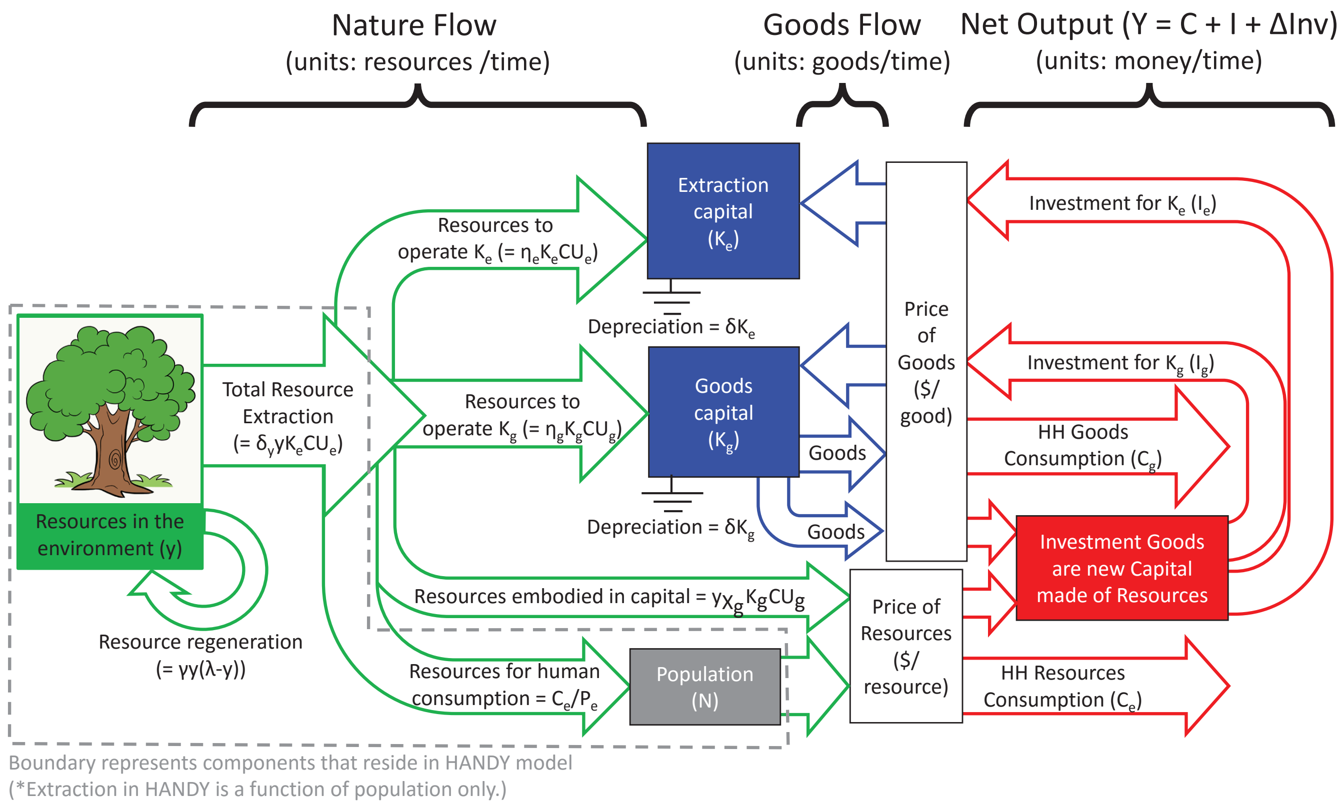


Figure 1: The HARMONEY model explicitly tracks flows of natural resources, goods, and money using prices to translate from physical units to money. Flows and stocks of resources are in green (left of diagram). Flows and stocks of goods are in blue (center of diagram). Flows of money are in red (right of diagram). HH = household.

of Equations 1 and 2. Output is constrained indirectly by natural resources that are required to operate capital and feed the population.

The expression for resource extraction multiplies a technological parameter δ_y by the remaining resource available for extraction, y. In contrast to the HANDY model, extraction can be a function of capital and its utilization rate, K_eCU_e, rather than only the population of laborers. Total goods output is capital multiplied by its utilization rate, but augmented by a constant capital to gross output ratio, ν_g. The effective capital to gross output ratio for extraction varies as a function of resources, $\nu_e = \frac{1}{\delta_y y}$, providing a feedback such that as natural resources are depleted, each unit of extraction requires an increasing quantity of extraction capital.

$$X_e = min\left\{\delta_y y K_e CU_e, L_e a_e\right\} \tag{1}$$

$$X_g = min\left\{\frac{K_g CU_g}{\nu_g}, L_g a_g\right\} \tag{2}$$

2.1.2 Natural Resource

Our natural resource is a regenerative stock, such as a forest, as in the HANDY model. In the HANDY model, the rate of change of natural resource in the environment is equal to resource regeneration minus gross extraction, where extraction is a function of labor. We keep this same framework in Equation 3, but use the gross nature extraction expression of Equation 1. Regeneration is a function of the maximum size of the resource, λ_y, the resource regeneration rate, γ, and the amount of natural resource in the environment, y. The maximum regeneration rate occurs when $y = \lambda_y/2$.

$$\begin{aligned} \dot{y} &= \text{regeneration - extraction} \\ \dot{y} &= \gamma y(\lambda_y - y) - min\left\{\delta_y y K_e CU_e, L_e a_e\right\} \end{aligned} \tag{3}$$

2.1.3 Intermediate Demands

Matrix **A** (Equation 4) is the Leontief technical coefficient matrix specifying the intermediate demands for the two-sector model. Element $a_{ij} = x_{ij}/X_j$ is the quantity of the intermediate input from sector i needed by sector j, x_{ij}, to produce one physical unit of sector j's gross output, X_j.

Table 1 summarizes derivations of the technical coefficients. The resources consumed per unit of extracted resources, a_{ee}, is a function of the remaining resources, y. As resources deplete, $y \to 0$ and $a_{ee} \to \infty$, but due to the conservation of mass the maximum a_{ee} is one (i.e., at $a_{ee} = 1$ the extraction sector consumes all of its own output just to operate its existing capital). Resources used by the goods sector are consumed for operation, a^o_{eg}, and embodied in physical goods output, a^I_{eg}. Resource inputs to operate capital are determined by parameters η_e and η_g that define the quantity of resource input per unit of capital operation at full capacity utilization. Symbol y_{X_g} is the resource input needed to become embodied in a physical unit of goods sector output. Each of η_e, η_g, and y_{X_g} has units of "resource per unit of capital per time." We assume a_{gg} and a_{ge} are constants (see Appendix).

$$\mathbf{A} = \begin{bmatrix} a_{gg} & a_{ge} \\ a_{eg} & a_{ee} \end{bmatrix} = \begin{bmatrix} \frac{x_{gg}}{X_g} & \frac{x_{ge}}{X_e} \\ \frac{x_{eg}}{X_g} & \frac{x_{ee}}{X_e} \end{bmatrix} \tag{4}$$

2.1.4 Capital accumulation, investment and debt

We use the perpetual inventory method for capital accumulation as physical investment minus physical depreciation occurring at constant rate, δ, for each sector (Equation 5). Physical investment in K_e and K_g is represented by I^g_e and I^g_g, respectively, where the superscript g indicates capital has units of goods. Capital investment in

Table 1: Equations for elements of technical requirements matrix, **A**.

a_{ee}	resources to operate extraction capital	$\frac{\eta_e K_e CU_e}{\delta_y y K_e CU_e} = \frac{\eta_e}{\delta_y y}$
a_{eg}	resources to operate goods capital and invest	$a_{eg} = a^o_{eg} + a^I_{eg}$
a^o_{eg}	resources to operate goods capital	$\frac{\eta_g K_g CU_g}{(K_g CU_g/\nu_g)} = \eta_g \nu_g$
a^I_{eg}	resources that become physical goods (incl. investment)	$\frac{y_{X_g} K_g CU_g}{(K_g CU_g/\nu_g)} = y_{X_g} \nu_g$
a_{ge}	goods input for extraction	constant
a_{gg}	goods input to produce goods	constant

money units for each sector, I_i, is the physical investment multiplied by the price of goods, P_g, such that $I_e = I^g_e P_g$ and $I_g = I^g_g P_g$.

$$\dot{K}_i = I^g_i - \delta K_i = I_i/P_g - \delta K_i \tag{5}$$

We model investment as a function of profit. Total investment in each sector is some proportion, $\kappa_{0,i}$, of depreciation added to a multiple, $\kappa_{1,i}$, of net profit, Π_i (Equation 6). This differs from Keen (2013) and Bovari *et al.* (2018) who use functions of profit rate and profit share, respectively. However, for constant depreciation, the investment parameters can be chosen to produce the same level of investment if based on profit rate or profit share (see Supplemental Section SI.1.8).

$$I_i = max\{0, \kappa_{0,i} P_g \delta K_i + \kappa_{1,i} \Pi_i\} \tag{6}$$

The change in private debt for each sector is the difference between net investment (gross investment minus depreciation) and net profit, Π_i (where Π_i already is gross profit minus depreciation per Tables S.1 and S.2). During simulation, in solving the change in debt using Equation 7, investment is based on the profit, price of goods, and capital from the last time period, but net profit is calculated for the current time period. (See description of lagged equations in Section 2.3.)

$$\dot{D}_i = I_i - P_g \delta K_i - \Pi_i \tag{7}$$

2.1.5 Wages and Labor

Labor in each sector earns the same wage per person, w. The change in the wage increases with the participation rate and the current wage through an increasing real-valued function of participation, $\phi(\lambda_N)$, as used in Goodwin's predator-prey business cycle model of employment and wages [40]. The function $\phi(\lambda_N)$ is the Phillips Curve, and for it we use the general exponential function of Keen (2013). While Keen (2013) models the change in wage by including additional factors than in Equation 8, we follow Bovari *et al.* (2018) in using the simpler Equation 8. See Supplemental Section SI.1 for a description of $\phi(\lambda_N)$.

$$\dot{w} = \phi(\lambda_N) \times w \tag{8}$$

The participation rate, λ_N (employment), is the labor of both sectors divided by population, N. We specify a maximum participation rate, $\lambda_{N,\max} < 1$, to represent that some fraction of the population is too young, old, or otherwise unable to work. If the total participation rate is equal to its maximum then labor is the limiting factor. In this case each sector's output is its labor times labor productivity, a_i, and capital capacity utilization decreases as needed to equate factors in output Equations 1 and 2 (see Supplemental Section SI.4).

$$\lambda_N = \frac{L_e + L_g}{N} \tag{9}$$

2.1.6 Cost of Production

The full cost of production per unit of gross output, c_i, is the sum of intermediate spending, wages, debt interest payments, and depreciation all divided by gross output (Equations 10 and 11). This defininition of the unit cost of production follows that of "normal average total costs" (NATC) in the post-Keynesian pricing text by Lee [42] as well as the defined cost in Bovari *et al.* (2018). The direct costs are intermediate costs and wages. The depreciation allowance cost is part of "shop expenses" [42]. Section 8.3 of Godley and Lavoie (2007) describes interest payments ($= r_L D_i$ with interest rate on loans, r_L) as part of full cost pricing that we pursue.

$$c_g = P_g a_{gg} + P_e a_{eg} + (wL_g + r_L D_g + P_g \delta K_g)/X_g \tag{10}$$

$$c_e = P_e a_{ee} + P_g a_{ge} + (wL_e + r_L D_e + P_g \delta K_e)/X_e \tag{11}$$

2.1.7 Prices

Equation 12 defines price of output from each sector, P_i, as a markup, $\mu_i \geq 0$, on cost.

$$P_i = (1 + \mu_i)c_i \tag{12}$$

We define price using two different assumptions for the markup. The first assumes the markup is constant. We combine each sector's price and cost equations (Equations 10–12) to derive Equations 13 and 14 that isolate prices.

$$\frac{wL_g + r_L D_g}{X_g} = \left(\frac{1}{1+\mu_g} - a_{gg} - \frac{\delta K_g}{X_g}\right) P_g - (a_{eg}) P_e \tag{13}$$

$$\frac{wL_e + r_L D_e}{X_e} = (-a_{ge} - \frac{\delta K_e}{X_e}) P_g + \left(\frac{1}{1+\mu_e} - a_{ee}\right) P_e \tag{14}$$

Placing these equations into matrix form (Equation 15) allows us to solve for prices using Equation 16 where P is the price vector, $\tilde{V}$ is a quasi-value added vector containing only wage and interest payment shares of value added, the Δ matrix represents the value of depreciating capital, $\hat{M}$ is a diagonal matrix containing the markup factors, and $\mathbf{A}^T$ is the transpose of the technical coefficients matrix, $\mathbf{A}$. See Supplemental Section SI.1 for more detailed price derivation.

$$\tilde{V} = (\hat{M} - \mathbf{A}^T - \Delta)P \tag{15}$$

$$P = (\hat{M} - \mathbf{A}^T - \Delta)^{-1}\tilde{V} \tag{16}$$

For comparison, we derive a second price equation that assumes a variable markup that is a function of profit rate, $\pi_{r,i}$. Profits for each sector, Π_i, can be defined based upon a markup applied to the full cost of production, c_i.

$$\Pi_i = \mu_i c_i X_i \tag{17}$$

Because of the definition of profit rate, profit is also defined as profit rate multiplied by the price of goods (capital) and the stock of capital:

$$\Pi_i = \pi_{r,i} P_g K_i \tag{18}$$

Equating Equations 17 and 18 leads to a variable markup on costs:

$$\mu_i = \frac{\pi_{r,i} P_g K_i}{c_i X_i} \tag{19}$$

Thus, prices are the same as in Equation 12, but with the variable markup from Equation 19.

$$P_i = (1 + \mu_i) c_i = \left(1 + \frac{\pi_{r,i} P_g K_i}{c_i X_i}\right) c_i = c_i + \frac{\pi_{r,i} P_g K_i}{X_i} \tag{20}$$

In the same manner as for the constant markup case, we combine the sector cost Equations 10 and 11 with the price Equation 20 to isolate prices as in Equations 21 and 22. Placing these equations into matrix form (Equation 23) allows us to solve for prices using Equation 24 where the bold **1** is the identity matrix, the Π_r matrix holds the profit rate factors, and all other matrices are the same as in Equation 16. See Supplemental Section SI.1 for more detailed price derivation.

$$\frac{wL_g + r_L D_g}{X_g} = \left(1 - a_{gg} - \frac{\delta K_g}{X_g} - \frac{\pi_{r,g} K_g}{X_g}\right) P_g - (a_{eg}) P_e \tag{21}$$

$$\frac{wL_e + r_L D_e}{X_e} = \left(-a_{ge} - \frac{\delta K_e}{X_e} - \frac{\pi_{r,e} K_e}{X_e}\right) P_g + (1 - a_{ee}) P_e \tag{22}$$

$$\tilde{V} = (\mathbf{1} - \mathbf{A}^T - \Delta - \Pi_r) P \tag{23}$$

$$P = (\mathbf{1} - \mathbf{A}^T - \Delta - \Pi_r)^{-1} \tilde{V} \tag{24}$$

2.1.8 Inventories and Capacity Utilization

Equations 25 and 26 show the rate of change of the physical quantity of inventory for goods, g, and wealth, w_H, respectively. We use the term "wealth" for the physical inventory of extracted resources to keep the nomenclature of the HANDY model [1]. Importantly this use of the term wealth is not the conventional economic meaning that refers to the value of a stock of assets. The change in physical inventory for each sector is the difference between the reference and current inventory coverage multiplied by the targeted physical consumption of each sector output. If the inventory coverage is higher than the set reference, then inventory decreases, and vice versa. In essence, the inventories scale up with demand. For the dynamics of wealth, or inventory of resources, this economic approach is different than in the HANDY model.

$$\begin{aligned} \dot{w}_H &= (\text{reference inventory coverage - inventory coverage})(\text{targeted consumption of resources}) \\ \dot{w}_H &= (IC_{ref,e} - IC_e)(C_e/P_e + a_{eg} X_g + a_{ee} X_e) \end{aligned} \tag{25}$$

$$\begin{aligned} \dot{g} &= (\text{reference inventory coverage - inventory coverage})(\text{targeted consumption of goods}) \\ \dot{g} &= (IC_{ref,g} - IC_g)((C_g + I_g + I_e)/P_g + a_{ge} X_e + a_{gg} X_g) \end{aligned} \tag{26}$$

Wealth and goods inventories can rise and fall with business cycles. We model capital capacity utilization (CU_i) as a function of perceived inventory coverage as in Sterman (2000) [43]. Perceived inventory coverage for sector i, $IC_{i,perceived}$, is defined as the quantity of physical inventory divided by a time delay and the targeted consumption for that sector output. The higher the total consumption for a given output, the larger the inventory stock needed to buffer one year of consumption, for example. See Supplemental Section SI.1 for inventory equations describing inventory coverage and capacity utilization and Section SI.4 for how we determine capacity utilization under resource and participation rate constraints described in Section 2.2.

2.1.9 Monetary Net Output and Consumption

In a multi-sector model, due to intermediate consumption each sector's value added does not necessarily equal its monetary net output. As indicated by Table S.2, the sectoral monetary gross output is equal to intermediate sales plus net output. Since we specify gross extraction and intermediate sales, we solve for monetary net output, Y, as in Equation 27.

$$Y = \hat{P}X - \hat{P}\mathbf{A}X = \hat{P}(\mathbf{1} - \mathbf{A})X \tag{27}$$

The value of inventories, INV, is equal to the current unit production cost times the physical quantity of inventory (Equations 28 and 29). The change in the value of inventory, ΔINV, is the current value of inventory minus the value from the previous time period. We model the value of inventory using lagged equations (see Section 2.3).

$$\text{INV}_g = c_g g \tag{28}$$

$$\text{INV}_e = c_e w_H \tag{29}$$

The present model is of a closed economy (no imports or exports) with no government. Then by convention, net monetary output is equal to final consumption plus investment plus change in value of inventories. We assume household consumption, C_i, is fully accommodating and is the residual left from subtracting investment and change in value of inventories from net output (Equations 30 and 31). Since the goods sector produces investment goods, there is no production of investment goods output from the extraction sector, and extraction sector net output is equal to sector consumption minus change in the value of inventory.

$$C_g = Y_g - P_g(I_g^g + I_g^e) - \Delta\text{INV}_g \tag{30}$$

$$C_e = Y_e - \Delta\text{INV}_e \tag{31}$$

2.1.10 Population

We follow the HANDY model in keeping birth rates, β_N, constant (for simplicity) and allowing the feedback in the population dynamics to be set by defining death rates as a function of physical consumption of extracted resources, $\frac{C_e}{P_e}$. This death rate function, $\alpha_N\left(\frac{C_e}{P_e}\right)$ (Supplemental Equation S.9, Section SI.1.4) decreases from a maximum value at zero resource consumption to a minimum positive death rate at some specified per captia resource consumption. The rate of change of the population, N, is births minus deaths (Equation 32).

$$\dot{N} = \beta_N N - \alpha_N\left(\frac{C_e}{P_e}\right) N \tag{32}$$

2.2 Biophysical Constraining Thresholds

Resource extraction is allocated among the operation of capital, household consumption, and resource embodied in investment (see Figure 1). It is possible that the extraction rate of resources is insufficient to fully satisfy minimum levels of household consumption and operational inputs for capital along with the desired level of investment. To account for output constraints based on labor or resource flows, the model dynamics operate within one of eight possible modes based on three binary threshold criteria (i.e., $2^3 = 8$). See Supplemental Section SI.4 and Figure S.2 for a description and decision tree of the algorithm for solving the model equations in each mode of operation.

We previously described the first threshold criterion, the maximum participation rate ($\lambda_{N,\max}$). By first assuming capital is the limiting factor in production, we calculate required labor and the participation rate. If that participation rate is above the threshold, then we decrease labor in each sector by the same percentage such that participation rate equals its maximum value. In this case gross output and capacity utilization decrease. The second threshold criterion is a minimum household consumption of resources per person ($\rho_e > 0$). If per capita household resource consumption ($= C_e/(P_e N)$) would otherwise be less than ρ_e, we set $C_e/(P_e N) = \rho_e$ and reduce physical investment to match total resource consumption to extraction. A reduction in investment reduces gross output of goods which in turn reduces total resource consumption. The third and final threshold criterion is a minimum household consumption of goods per person (ρ_g) that we set to zero in this paper. If this threshold is met, investment is reduced, as needed, possibly to zero, in which case intermediate demands account for all goods consumption.

2.3 Lagged equations for simulation

In the real world, data are only available for decision making after some amount of time. For example, firms and governments know profits and net output from the previous year, but they generally don't know those values for last month, yesterday, or the previous hour. To make certain variables available within the simulation code, we model their values from the "previous time period" as a first order lag (see Supplemental Section SI.1.6 and Equation S.13) [43]. Some similar macroeconomic models also use lagged states for core economic information states, and the practice is normal within system dynamics models. For instance Keen (2013) uses lags for modeling household and firm bank deposits, and both Keen (2013) and Bovari *et al.* (2018) do so for calculating inflation. For each sector i, we update the following variables using a first order lag: capacity utilization (CU_i), perceived inventory coverage ($IC_{i,perceived}$), price (P_i), net output (Y_i), profit (Π_i), value added (V_i), and value of inventory (INV_i). For further reference see Supplementary Section SI.8 that lists every differential equation in the model.

2.4 Endogenous and Exogenous Variables

Table 2 lists the endogenous and exogenous variables included in the model. Appendix Table A.2 lists the chosen values for all model parameters that describe endogenous and exogenous variables.

Table 2: A list of endogenous and exogenous variables within the model.

Endogenous Variables	Exogenous Variables
population (via death rates)	birth rates
prices	parameters defining the natural resource
wages	wage function parameters
debt (of firms)	interest rates
capacity utilization	labor productivity
inventories	goods sector productivity (capital:output ratio, ν_g)
economic output	resource requirements to make capital (y_{X_g})
physical output	resource requirement to operate capital (η_g, η_e)
investment	time delays
capital	
labor	
household consumption	
net power ratios (e.g., NEPR)	

2.5 Net Power and Energy Accounting

In addition to analyzing the collective model variables, we also consider system-wide net energy metrics that summarize complex interactions between the energy sector with the rest of the economy. These metrics are used by biophysical scientists and economists, but they are uncommon within orthodox economics. They enable further understanding of the model's construction and structural dynamics. We hope these metrics enable different communities of scientists and economists to explore similar economic phenomena from different perspectives.

Anthropologist Joseph Tainter summarizes the motivation for using net energy metrics. He posits that the "energy-complexity spiral" describes a feedback in which societies increase complexity to solve social problems, but that an increase in resources is required to enable this complexity [44, 45, 46]. Further, he and others, including ecologists such as Howard T. Odum [47] and Charles Hall [48], hypothesize that the net energy, or energy gain, from extracting a resource indicates the potential for change: "Energy gain has implications beyond mere accounting. It fundamentally influences the structure and organization of living systems, including human societies." [49]

Energy analysis became a field of study in the mid-1970s. There was a new need to better understand how energy is used in society. This activity was triggered by peak U.S. oil production in 1970, the Arab Oil Embargo of 1973, and the 1974 increase (of over one-hundred percent) in the posted price of oil by the Organization of Petroleum Exporting Countries (OPEC). The International Federation of Institutes for Advanced Study (IFIAS) organized two energy analysis workshops in Sweden, one each in 1974 and 1975. The workshops described a major objective of energy analysis as "the evaluation of resource flows in societal processes using physical units." [**?**] For approximately a decade, energy analysts derived methods to study the energy requirements of economic sectors and technologies, including technologies whose purpose is to extract energy resources. Analysts estimated the net energy of extraction technologies as the energy extracted minus the energy requirements to build and operate the technology. Net energy analysis lost favor in the 1980s as oil prices plunged. It regained prominence in the 2000s for evaluating the life cycles of liquid biofuels, then mandated for certain consumption levels in the United States and European Union. Researchers have speculated on minimum levels of net energy "required" to sustain society [48], and calculation of net energy metrics within our model allows a theoretical and internally-consistent investigation of their relation to economic growth, structure, and population.

King *et al.* (2015) point out that net energy metrics broadly fall into two categories: power return ratios (PRR) and energy return ratios (ERR). PRR metrics divide some power output from an energy system by a quantity of power input required to operate the energy system under investigation. ERR metrics are similar, but the numerator and denominator are the life cycle energy outputs and inputs, respectively (i.e., energy is power integrated over time). These ratios characterize the power (or energy) generated by the energy sectors relative to the power (or energy) consumed by the energy sectors themselves. This self-consumption can include both operating inputs and energy embodied in investment.

We define two PRRs: the net external power ratio (NEPR) in Equation 34 and net power ratio (NPR) in Equation 35. NEPR is net resource extraction divided by the extraction sectors' own use of resources. NEPR represents what is often termed "energy return on energy invested" (EROI) in much of the literature [50]. Since our PRR calculations use instantaneous resource flow rates in the numerator and denominator, the term *power* return ratio is more appropriate than energy return ratio. However, much of the net energy literature uses the terminology EROI and ERR to refer both to ratios of energy and ratios of power.

To calculate NEPR, we calculate the resources embodied in extraction capital, K_e, via a resource intensity, ϵ, that measures gross resources extraction required per net physical output of each sector (see [51], [52], and [53] for the methodology for calculating "energy intensities"). In Equation 33, $\hat{y}_{\text{extract}}$ is a 2×2 diagonal matrix with non-zero elements *only* for the gross extraction by sectors that extract resources from the environment. For our model $\hat{y}_{\text{extract},ee} = \hat{y}_{\text{extract},22} = X_e$, $\hat{y}_{\text{extract},gg} = \hat{y}_{\text{extract},11} = 0$, $\hat{X}$ is a diagonal matrix of the gross physical output of each sector (X_g and X_e), $\mathbf{1}$ is the identity matrix, $\mathbf{A}$ the technical coefficients matrix, and matrix $\mathbf{E}$ is a 2×2 matrix of resource intensities, ϵ_{ij}. The first row of $\mathbf{E}$ becomes zero, and the second row contains the resource intensities of ϵ_{eg} representing the gross resource input per unit of net physical goods output, $\frac{Y_g}{P_g}$, and ϵ_{ee} representing the gross resource input per unit of net physical resources output, $\frac{Y_e}{P_e}$. Thus, the embodied resources in extraction capital at each time step is equal to $\epsilon_{eg}\frac{I_e}{P_g}$. Recall that $X_e = \delta_y y K_e CU_e$ as well as that $\frac{I_e}{P_g}$ has units of goods and represents the physical goods allocated to become new extraction capital.

$$\begin{bmatrix} 0 & 0 \\ \epsilon_{eg} & \epsilon_{ee} \end{bmatrix} = \mathbf{E} = \hat{y}_{\text{extract}} \hat{X}^{-1} (\mathbf{1} - \mathbf{A})^{-1} \tag{33}$$

$$\begin{aligned} NEPR &= \frac{\text{resource extraction} - \text{resources required to invest in } K_e - \text{resources required to operate } K_e}{\text{resources required to invest in } K_e + \text{resources required to operate } K_e} \\ &= \frac{X_e - \epsilon_{eg}\frac{I_e}{P_g} - a_{ee}X_e}{\epsilon_{eg}\frac{I_e}{P_g} + a_{ee}X_e} \end{aligned} \tag{34}$$

Per King *et al.* (2015) we can consider an economy level net power ratio ($\text{NPR}_{\text{economy}}$) as similar to but distinct from NEPR that more specifically describes the extraction sector [50]. With only one extraction ("resources") sector, $\text{NPR}_{\text{economy}}$ is calculated using only ϵ_{ee} in Equation 35. Economy-wide gross power ratio ($\text{GPR}_{\text{economy}}$) equals one plus $\text{NPR}_{\text{economy}}$ [54], and it has been referred to as the EROI of the economy [55]. The upper limit for $\text{NPR}_{\text{economy}}$ is defined using a_{ee} as $\text{NPR}_{\text{economy, upper limit}} = \frac{1-a_{ee}}{a_{ee}}$, where $1 - a_{ee}$ is the fraction of extracted resources left for all other economic activity after operating extraction capital, but not yet accounting for investment in extraction capital [51]. As such, $\text{NPR}_{\text{economy, upper limit}}$ is largely defined by the resource efficiency of extraction sector capital.

$$NPR_{\text{economy}} = \frac{1}{\frac{\text{gross power extracted}}{\text{net power output}} - 1} = \frac{1}{\epsilon_{ee} - 1} \tag{35}$$

2.6 Limitations

There are many limitations of the HARMONEY model described in this paper. The limitations are of two general varieties: neglected components (sectors, variables) and simplified behavior.

Models, by definition, are simplified representations of the real world, and hopefully useful. The present model excludes many features of a modern economy that could provide additional insight but also additional complexity. Some missing features are a government sector (along with tax and redistribution policy) and additional countries with which to have trade. There is no central bank or assumed behavior to adjust interest rates, and for simplicity, the model assumes no retained bank profits.

Our model omits variables related to environmental resources and sinks, such as land, greenhouse gas emissions, and types of pollution that do not directly contribute to or limit economic production. Additionally, future versions could model several types of natural resources to simultaneously represent renewable flows (e.g., wind and solar insolation), renewable stocks (e.g., forests), and nonrenewable stocks (e.g., minerals as well as fossil fuels) to inform the debate on the rate and impact of a low-carbon energy transition.

The model also simplifies many behavioral aspects. More realistic population demographics could be included by modeling multiple population ages or using birth and death rates reflecting the demographic transition hypothesis, such as in Uehara *et al.* (2015). We do not model household consumption behavior (e.g., consumption as a function of disposable income) as is common in post-Keynesian approaches. The consequence is that all wages and bank net interest go to households, and we don't model household debt (see transactions Table S.1 in Supplemental Section SI.2). While the model informs distribution between wages, interest income, and profits, it does not inform what percentage of the population earns income via wages and interest income.

To focus on the core model dynamics, we avoid exogenous assumptions to model technical change and possible substitution among labor, capital, and resources. Naqvi and Stockhammer (2018) present a post-Keynesian model, with endogenous technical change that affects labor, capital, and resource productivity, in which they compare results using a Leontief and constant elasticity of substitution production function [56]. Future work could perform a similar comparison with the present model, and we could also model technological change where investment improves factors describing resource inputs to capital operation (η_i), goods sector capital output (ν_g), and labor productivity (a_i).

3 Results

In describing the trends of the simulated scenarios, we focus on describing which variables increase and decrease together and when trend changes occur. Because the model sets a minimum threshold on household consumption for both resources and goods, as well as a maximum employment rate, many of the important trends occur in the context of whether one or more of these thresholds are binding. We describe "normal growth" as the condition when none of these three threshold constraints are binding. That is to say, normal economic growth occurs when household consumption is not constrained (it might be increasing or decreasing) and a higher fraction of the population could be employed by investing in more capital.

In each simulation, all model parameters and initial conditions are the same unless otherwise indicated. See Appendix Table A.2 for a list of parameter values and Appendix Table A.3 for initial conditions.

We arrange the Results section as follows. The first subsection describes the model trends for each of three general scenarios (Renewable-Low, Renewable-High, and Fossil) when assuming a constant cost markup. These descriptions indicate which of the three threshold criteria are binding (see Sections 2.2 and SI.4). The second subsection summarizes important insights from the constant markup simulations. The third and final subsection compares results between the constant and variable markup assumption.

3.1 Constant Markup

3.1.1 Simulation Scenarios

We simulate three general scenarios when assuming prices derive from a constant markup on full costs. One is a "fossil" resource scenario defined by a resource regeneration rate of zero ($\gamma = 0$) in Equation 3. The other two are "renewable" scenarios with a positive resource regeneration rate, $\gamma = 0.01$. Two dynamic patterns occur for the renewable resource scenarios. The rate of accumulation of capital, or the rate of investment, relative to population is the distinguishing characteristic that delineates the two patterns. Thus, we define each renewable scenarios as one with a "low" and "high" rate of investment. The difference depends upon whether the combined effect of the cost markup and the two parameters of the investment function create a relatively low or high level of gross investment. We display results for Fossil, Renewable-Low, and Renewable-High investment scenarios using dotted, dashed, and solid lines, respectively, in Figures 2 and S.4.

United States (U.S.) data create context for how investment might vary relative to profits, and they show gross private domestic investment is usually 50% greater or more than net profits (see Figure S.3). These data guide our scenario definitions based on changes to the investment function parameters, κ_0 and κ_1, and the cost markup, μ. Table 3 indicates the parameter values that vary to define the scenarios. Each of the three Renewable-Low and Renewable-High investment scenarios (a-c) produces the same general dynamic result. The dynamics of each renewable scenario can be achieved by varying only one of the three investment or cost markup parameters because each one affects the rate of investment. For example, if the markup is lower, then one can increase one of the investment function parameters to achieve the same model dynamics. To make the Fossil scenario comparable to Renewable scenarios, we choose the parameters for the Fossil scenario to approximately match the initial growth dynamics of the Renewable-High scenario. Thus, we increase the maximum available nature by an order of magnitude to $\lambda_y = 1000$. Because the extraction rate of natural resources is proportional to $\delta_y y$, we also decrease δ_y by an order of magnitude in the Fossil scenario so that the product $\delta_y \lambda_y = (0.012)(100) = (0.0012)(1000) = 1.2$ is the same for the Fossil and Renewable scenarios. For clarity of discussion in the main text, we discuss the dynamics of each renewable pattern only for the two Renewable scenarios labeled with (a), and Figure 2 displays those results. Supplemental Figure S.4 shows results for all six renewable scenarios listed in Table 3. In effect there is an infinite combination of values for the parameters that achieve the same two general dynamic trends.

Table 3: The Low and High investment scenarios are defined by a low or high value, respectively, for either one of the investment function parameters or the cost markup. The parameters values are the same for both sectors.

Scenario	Investment Parameters (Equation 6) $\kappa_{0,i}$	$\kappa_{1,i}$	Cost Markup (Equation 16) μ_i	Extraction Factor (Equation 3) δ_y	Max. Resource Size (Equation 3) λ_y	Regeneration Rate (%) (Equation 3) γ
Renewable-Low Invest (a)	1.0	1.3	0.07	0.012	100	1%
Renewable-Low Invest (b)	1.3	1.0	0.07	0.012	100	1%
Renewable-Low Invest (c)	1.015	1.015	0.09	0.012	100	1%
Renewable-High Invest (a)	1.0	1.5	0.07	0.012	100	1%
Renewable-High Invest (b)	1.5	1.0	0.07	0.012	100	1%
Renewable-High Invest (c)	1.015	1.015	0.11	0.012	100	1%
Fossil	1.0	1.55	0.07	0.0012	1000	0%

3.1.2 Model Simulation: Renewable - Low Investment

When investment is relatively low, the model simulates "normal growth" followed by a rapid collapse. Two sequential modes of operation describe the evolution of the Renewable-Low investment scenario. The times discussed in the bullets below refer specifically to results in Figure 2, but the sequence of events is the same for all Renewable-Low scenarios (see Figure S.4).

- **Mode 1 (T=0 to T=69), "normal growth":** The quantity of capital, not labor, constrains economic output. Capital, population, and per capita consumption of household physical goods all increase. Per capita household consumption of physical resources first increases before declining after T=48.
- **Mode 2 (T=69 to T=74), Household resource consumption constraint:** Household resource consumption per person reaches its minimum level ($\rho_e > 0$, Figure 2(p)) at T=69, and this event triggers this second mode of operation. Goods sector output, including investment in goods capital, K_g, reduces to prioritize resource allocation to people (household resource consumption) forcing its capacity utilization to decline (Figure 2(o)). NEPR (including resources to both operate and invest in K_e) and household goods consumption approach zero at the end of this mode as extracted resources are increasingly consumed by the extraction sector itself (Figure 2(l) and (q)). Investment also declines rapidly to prioritize household resource consumption as well as operation of existing capital (Figure 2(r)). By T=74, the system has collapsed at NEPR=0.

Perhaps counterintuitively, the rapid collapse triggered during Mode 2 occurs because capital does not accumulate fast enough during Mode 1 to enable labor to reach the maximum participation rate ($\lambda_{N,\max} = 80\%$) before the extraction sector reaches a peak extraction rate. We interpret the system reaching a resource constraint as both reaching peaks in total and per capita extraction rates (Figure 2(b) and (m)) and the net external power ratio (NEPR) not stabilizing above zero (Figure 2(l)). Importantly resource flow constraints impact economic outcomes even during "normal growth" of Mode 1, before reaching the threshold per capita resource consumption, ρ_e. The resource price (Figure 2(h)) rises rapidly after the peak in per capita resource extraction rate (at T$\sim$50). This price increase drives rapidly increasing profits and corresponding declines in wage and wage share. With rising profit, and because investment is curtailed in Mode 2 due to the constraint in flow of physical resources, firms rapidly repay debt, and the debt ratio declines (Figure 2(f)).

3.1.3 Model Simulation: Renewable - High Investment

When investment is relatively high, the system reaches the maximum participation rate before resource depletion forces a physical constraint of output near NEPR = 0 as occurs for the Renewable-Low scenario. After reaching

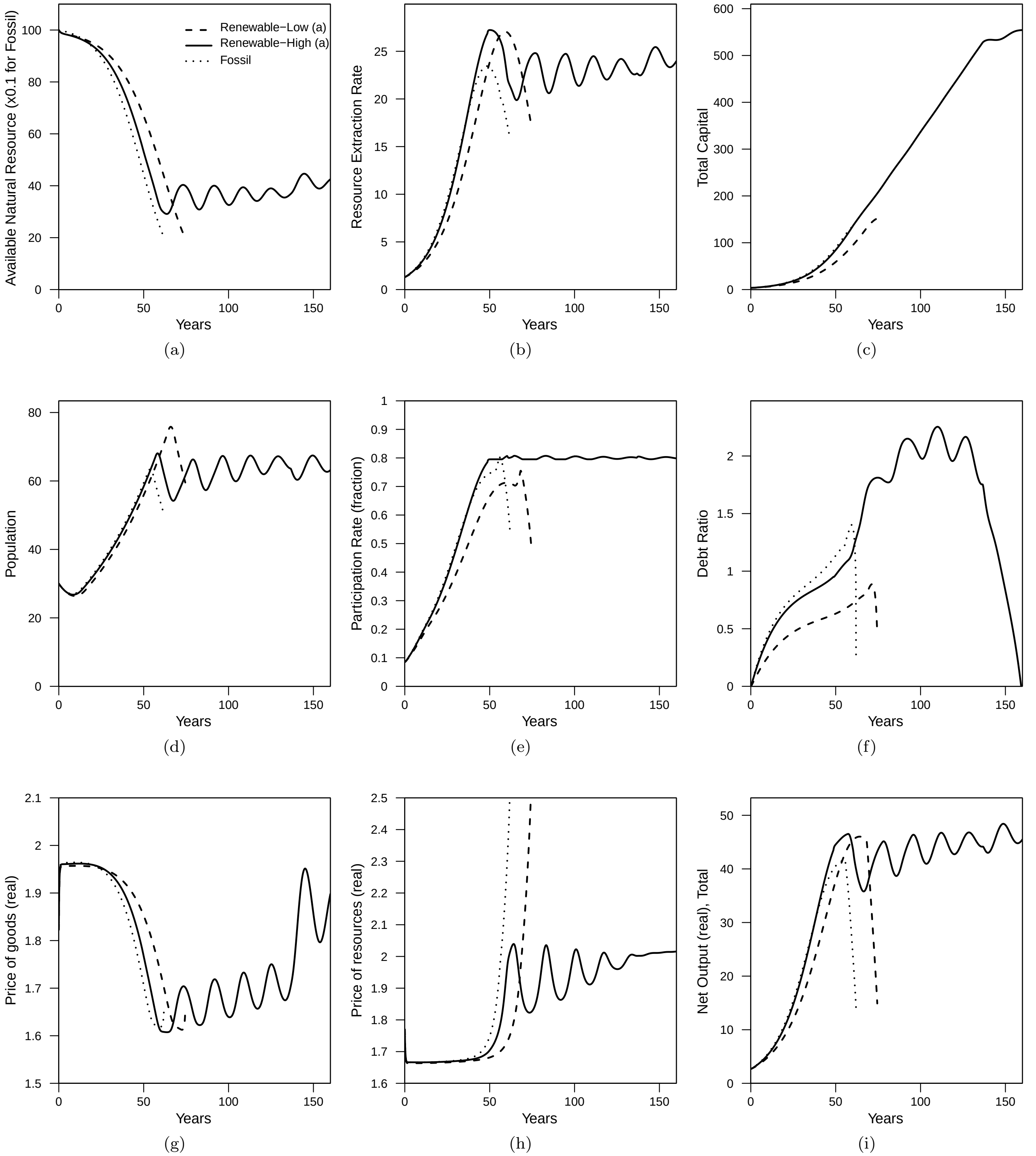


Figure 2: Simulation results when setting prices via a constant markup ($\mu_i = 0.07$) with $\kappa_0 = 1.0$ and a Low investment level using $\kappa_1 = 1.3$ (dashed line) versus a High investment level using $\kappa_1 = 1.5$ (solid line). (a) available resources (in the environment), (b) resource extraction rate, (c) total capital, (d) population, (e) participation rate, (f) debt ratio, (g) real price of goods, (h) real price of extracted resources, (i) total real net output, and (continued) ...

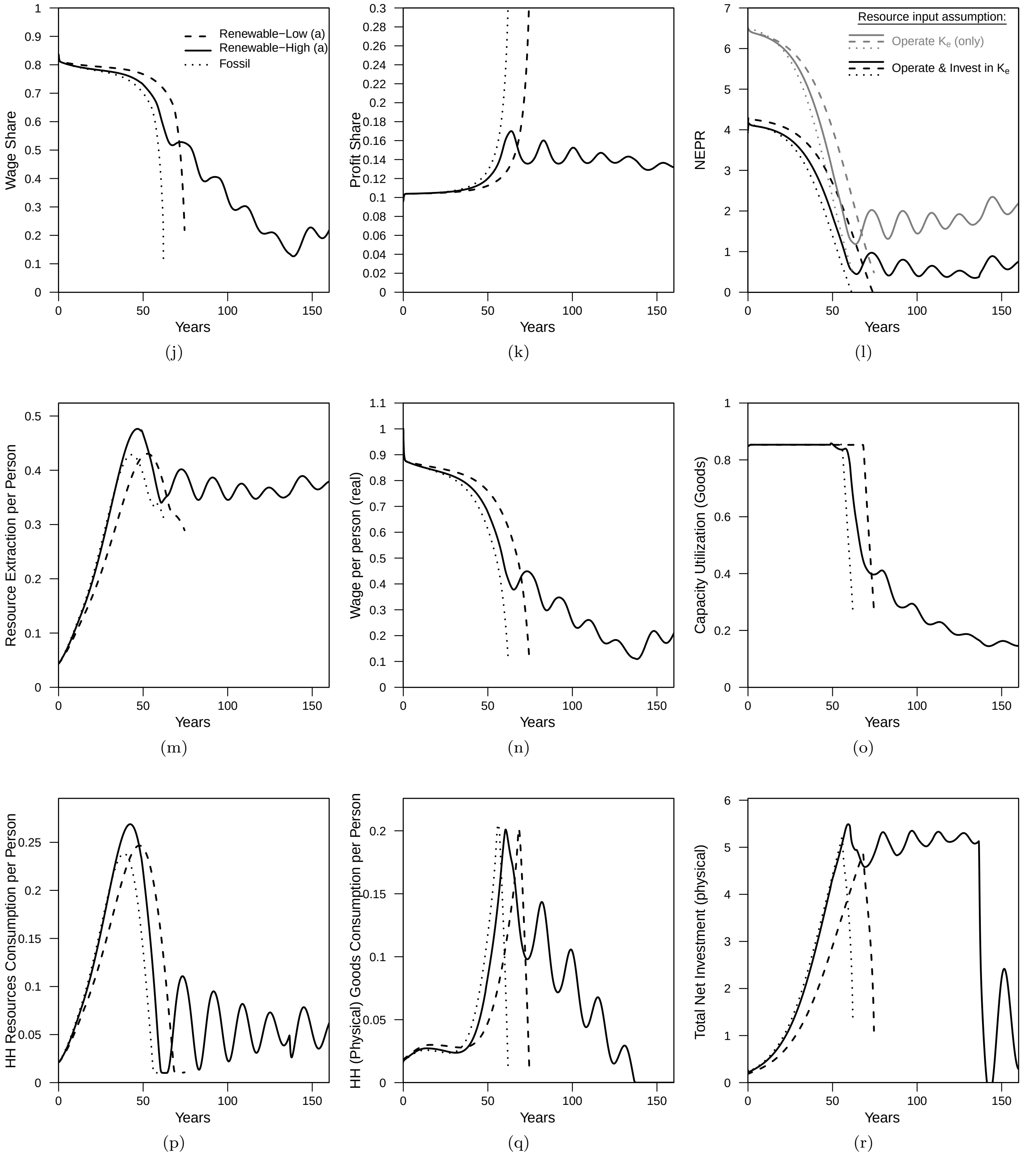


Figure 2: (continued) ... (j) wage share, (k) profit share, (l) net external power ratio (NEPR), (m) resource extraction per person, (n) real wage per person, (o) capacity utilization of goods capital, (p) household consumption of (physical) resources per person, (q) household consumption of (physical) goods per person, and (r) physical net investment.

a condition with labor as the factor-limiting input to economic growth, the system avoids rapid collapse after reaching peak per capita resource extraction. However, the response is not structurally neutral because wage share declines. Wage share declines in the short term because debt ratio increases, thus so does interest share. It declines over the longer term as the capital stock continues to increase, thus so does the depreciation share. Five sequential modes of operation describe the evolution of the Renewable-High scenario.

- **Mode 1 (T=0 to T=50), "normal growth":** The quantity of capital, not labor, constrains economic output. Capital, population, and per capita consumption of household physical goods all increase. Per capita household consumption of physical resources first increases before declining after T=43 (Figure 2(p)). Wage share is approximately constant (Figure 2(j)).
- **Mode 2 (T=50 to T=61), labor constrains output:** Participation rate reaches its maximum level, at T=50 (Figure 2(e)), triggering this second mode soon after peak per capita resource extraction (Figure 2(m)). Goods output and capacity utilization must slightly decline (Figure 2(o)) because there is not enough labor to operate capital at the target capacity utilization of 85%. Population peaks at T=58 and begins to decline (Figure 2(d)). Wage share now begins to decline (Figure 2(j)).
- **Mode 3 (T=61 to T=64), household resource consumption resides at minimum:** For a short time, per capita household consumption of resources resides at its minimum level ($\rho_e > 0$) (Figure 2(p)). Capacity utilization of both capital stocks declines rapidly to allocate an increasing fraction of net resources to households (Figures 2(o) and S.4(s)). NEPR stops its initial rapid decline (Figure 2(l)). Population declines enough such that the end of this mode is marked by per capita household consumption increasing above its minimum threshold.
- **Mode 4 (T=64 to T=137), household goods consumption declines to minimum:** Per capita household resource consumption is now increasing, and it oscillates above its minimum threshold level (Figure 2(p)). However, household goods consumption declines to its minimum level ($\rho_g = 0$, Figure 2(q)). Real net investment no longer increases and is approximately constant (Figure 2(r)). NEPR (including resources to operate *and* invest in K_e) remains relatively constant, but NEPR rises when considering only operational inputs to K_e (Figure 2(l)). Wage share continues to decline at an approximately constant rate per year (Figure 2(j)).
- **Mode 5 (T=137 to T=160), investment declines:** Household goods consumption remains at zero. Investment declines to reduce resource consumption of the goods sector and enable the extraction sector to continue operations (Figure 2(r)). Debt ratio declines because firms repay debt as physical resource flow constraints curtail investment below profits. This reduces interest payments and allows wage share to increase.

Compared to the Renewable-Low scenario, higher investment during the normal growth mode of the Renewable-High scenario directs more resources to capital leaving less for household consumption. Thus, the Renewable-High scenario has a lower peak population. After peaking, the population declines enough to enable household consumption to rebound above the minimum per capita resource consumption rate after T=64 (end of Mode 3). Similar to the Renewable-Low scenario, resource price rises rapidly after reaching the peak per capita resource extraction rate, but the rise is limited and stabilizes. This price pattern is similar to the real price of oil that averaged a "low" 19 $2015/BBL from 1881–1973 and then a "high" 58 $2015/BBL from 1973–2015 [57]. Stable resource and goods prices lead to relatively stable profit shares.

The Renewable-High scenario exhibits stable (but oscillating) population and resource stocks after Mode 3, and during Mode 4 (T=64 to T=137) gross investment outpaces depreciation such that the capital stock increases. Thus, during Mode 4 the value of capital depreciation increasingly dominates the share of value added (Figure S.4(x)), and real wages and wage share decline in response (Figures 2(n) and (j)). Similar to the Renewable-Low scenario, resource constraints eventually curtail physical investment (during Mode 5) such that net investment declines close to zero (Figure S.4(r)) and firms repay all private debt even to the extent of acquiring negative debt after T=160 (not shown in Figure 2(f)). The simulation of negative debt (after resource constraints) is a consequence of pricing behavior using a large enough constant markup. Several feedbacks could prevent this negative debt outcome, and exploration of these feedbacks is beyond the scope of this paper. However, the variable cost markup (Equation 19) provides one feedback that avoids negative debt by driving the cost markup to low levels. We discuss the variable cost markup results in Section 3.3 and Supplemental Section SI.7.

3.1.4 Model Simulation: Fossil

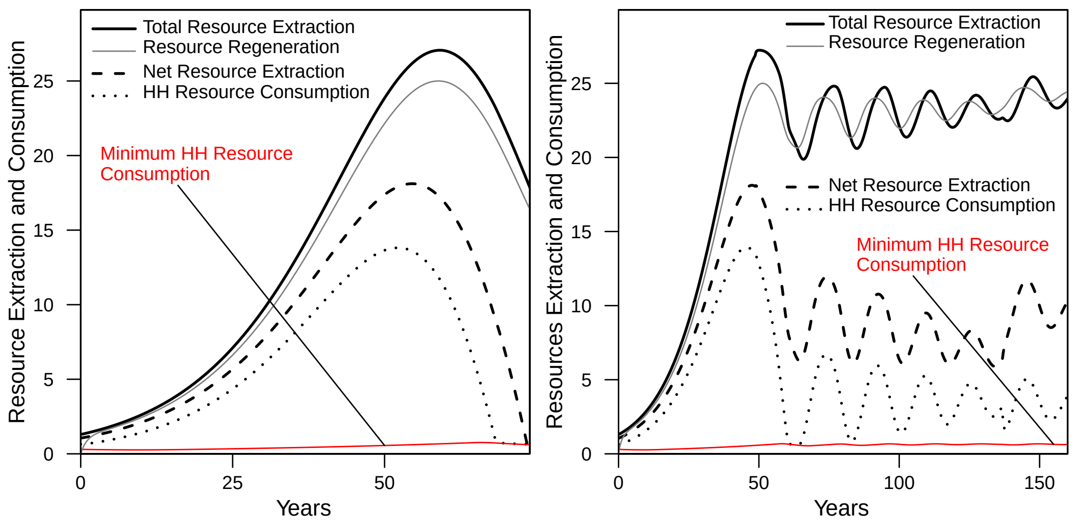
The trends of the Fossil scenario are similar in practically all respects to the Renewable-Low investment scenario. This particular Fossil scenario very briefly reaches maximum participation rate, but the general trend of rise and collapse occurs whether or not the model reaches maximum participation rate or not for any extended period of time. The simulation produces intuitive results given the definition of the Fossil scenario as extracting a finite resource that does not regenerate. For a description and simulation of a Fossil scenario using the variable cost markup, see Section SI.7 and Supplemental Figure S.5.

- **Mode 1 (T=0 to T=55), "Normal growth":** The quantity of capital, not labor, constrains economic output. Capital, population, and per capita consumption of household physical goods all increase. Per capita household consumption of physical resources first increases before declining after T=39 (Figure 2(p)).
- **Mode 2 (T=55 to T∼56), labor constrains output:** For a brief period, the simulation reaches the maximum participation rate, and labor constrains output (Figure 2(e)). Per capita household resources and goods consumption remain above their minimum levels (Figures 2(p) and (q)). Goods output and capacity utilization must decline slightly (Figure 2(o)) because there is now not enough labor to operate capital at the target capacity utilization of 85%.
- **Mode 3 (T∼56 to T∼57), household resource consumption constraint:** Household per capita resources consumption reaches its minimum threshold level (Figure 2(p)). Both investment (Figure 2(r)) and goods sector output are curtailed to prioritize resource allocation to households. These changes force a rapid reduction in goods sector capacity utilization (Figure 2(o)). Population has declined enough to reduce participation rate below its maximum level.
- **Mode 4 (T∼57 to T=62), household goods consumption declines to zero:** NEPR (including resources to both operate and invest in K_e) approaches zero at the end of this mode as the extraction sector increasingly consumes more of its own product (Figure 2(l)). Both household goods consumption and total investment capital decline rapidly under increasingly scarce net resource flows from the extraction sector (Figure 2(q)). By the end of this mode, the system has collapsed.

3.2 Avoiding Full Rapid Collapse in Constant Markup Scenarios

There are two basic dynamic trends from the constant markup scenarios. The trend of the Renewable-Low scenarios exhibit an initial period of growth followed by a full and rapid collapse after reaching a peak in the resource extraction rate. The collapse occurs to the point that resources are extracted to a level at which the extraction sector consumes 100% of its own output at NEPR = 0 (Figure 2(l)). The second trend of the Renewable-High scenarios also exhibit an initial growth period. However, the period following the peak extraction rate is not one of full collapse, but one with relatively constant net output (Figure 2(i)), slowly declining wage share (Figure 2(j)) with increasing depreciation share (Figure S.4(x)), and debt ratio that exhibits a rise and decline (Figure 2(f)).

Capital and population require natural resources to accumulate, function, and maintain themselves. This context helps explain the difference between the High and Low scenario dynamics as one that centers on population size when transitioning to a time period characterized by minimum per capita household resource consumption. Thus, the rate of investment before the transition determines initial conditions for dynamics after the transition. Higher investment allocates more resources to capital instead of population causing an earlier and lower population peak because more resources become embodied in capital relative to population, and vice versa. The Renewable-High scenario (peak population $N = 68$) avoids rapid collapse because population does not increase as high as in the Renewable-Low scenario (peak population $N = 76$). In effect, full collapse occurs in the Renewable-Low



(a) Renewable-Low(a) Scenario.

(b) Renewable-High(a) Scenario.

Figure 3: Four time series help envision why full collapse occurs in the (a) Renewable-Low(a) but not in the (b) Renewable-High(a) scenarios. Plotted are total resource extraction, resource regeneration, net resource extraction output from the extraction sector, household (HH) consumption of resources (in physical units), and a running calculation of the minimum level requirement for household consumption ($= N\rho_e$).

scenario because the system reaches a point at which there is not enough net resource extraction to provide the required minimum resource consumption for a population that is declining but still "too large".

We use Figure 3 to explain the causal process for reaching the full rapid collapse of the Renewable-Low scenarios. Net resource extraction is equal to total resource extraction minus all resources consumed by the extraction sector, both for operating and investing in new extraction capital. Household resource consumption, in units of nature per time (not money), is the residual resource available after meeting all other resource demands, including those for the goods sector. Once the system is constrained at the minimum household resource consumption threshold, resources extraction is already on the decline, and the system must prioritize household resource consumption by reducing goods sector demand for resources by lowering capacity utilization. As net resource extraction declines toward the level of minimum required household resource consumption, goods net output declines to zero. Eventually, the system reaches NEPR = 0, below which it cannot proceed because the net resource flow is too low to meet the minimum household consumption.

To prevent full collapse of the Renewable-Low scenarios, one could ask why the output of the goods sector could not decline even more rapidly (as indicated by goods sector decline in capacity utilization, CU_g) to accommodate allocation for minimum household resource consumption. The delay time in the capacity utilization, $\tau_{CU,g}$, dictates the rate at which goods sector output declines to accommodate allocation of resources for the minimum household resource consumption ($= N\rho_e$). However, even a shorter delay time ($\tau_{CU,g} > 0$) doesn't prevent the overall dynamic from being possible at some lower investment level. Another apparent tactic to avoid full collapse might be the reduction in extraction sector output, with corresponding reduction in capacity utilization, such that the available resource, y, regenerates to a larger size. At a larger natural resource level, the resource is extracted using fewer operational resource inputs (i.e., NEPR and $\text{NPR}_{\text{economy}}$ are larger), and steady state extraction can be maximized and equal to the maximum regeneration rate when $y = \lambda_y/2 = 50$. Thus, a higher net resource extraction would seemingly be available for household consumption. However, the problem with this tactic is that when the Renewable-Low(a) scenario reaches the minimum household resource consumption constraint (at T=69), the population is already too large and does not decline fast enough to prevent any reduction in total extraction from violating this constraint. Figure 3(a) shows that for the Renewable-Low(a) scenario, total resource extraction is always greater than resource regeneration. If one lowers the total resource extraction rate after T=69, the net resource extraction rate declines faster than that already shown and reaches full collapse (NEPR = 0) even sooner. A faster decline in population could allow the system to reduce resource extraction enough to enable the resource to regenerate to a stable level. However, there would still exist another lower investment scenario that exhibits the same trend of full collapse.

The Renewable-High(a) scenario avoids full collapse because it reaches the minimum per capita household resource consumption condition (at T=61) at a lower population, $N = 64$. The population declines enough by the end of Mode 3 (at T=64), to $N = 58$, such that the system provides per capita resource consumption higher than the minimum level for a smaller population. Further, starting at the end of Mode 2, population and capacity utilization of both sectors start to decline in tandem. Resource demands decline enough such that resource regeneration becomes larger than total resource extraction (at T=64 in Figure 3(b)). In the Renewable-Low(a) scenario, capacity utilization could not decline with population because a lower quantity of capital was trying to serve a higher level of minimum consumption of a larger population. After reaching the peak extraction rate, it is not necessarily the ratio of capital to population that determines collapse, but the absolute size of the population relative to possible net extraction rate. Further, while the Renewable-High scenarios reach the maximum participation rate, this is not a sufficient condition to avoid full collapse. Scenarios with intermediate investment rates between the defined High and Low scenarios of Table 3 do temporarily reach maximum participation rate before exhibiting full collapse.

There are many other model parameters that, if changed, can change the simulation outcome from the pattern of the Renewable-High to the Renewable-Low case, or vice versa. Here we list changes in parameters that, if starting with those in the Renewable-High(a) scenario, trigger it to the full rapid collapse pattern of the Renewable-Low scenarios. Because the Renewable-High(a) scenario resides near the threshold between the two dynamic patterns, very small changes to parameter values can trigger the rapid collapse pattern. The full rapid collapse occurs by varying $\mu_g = \mu_e$ from 0.07 to 0.06, δ_y from 0.012 to 0.013, λ_y from 100 to 101, δ from 0.03 to 0.017, a_{gg} from 0.1 to 0.09, a_{ge} from 0.20 to 0.21, ν_g from 1.5 to 1.6, β_N from 0.03 to 0.031, $\lambda_{N,\max}$ from 0.80 to 0.81, $\eta_g = \eta_e$ from 0.16 to 0.15, and ϕ_{min} (minimum rate of wage decline at no employment) from -0.05 to -0.002. All of these changes affect the growth rates and/or timing of changes to growth rates for population relative to capital. Some parameter changes have obvious impacts (e.g., a higher the birth rate, β_N, directly increases population growth relative to capital). Some induce less obvious interactions (e.g., higher maximum participation rate, $\lambda_{N,\max}$, triggers later reduction in capacity utilization and there is less net resource extraction available to sustain the population at its minimum consumption level before population declines enough). Changing these parameters in the opposite direction produces a similar pattern as the Renewable-High result, but with lower growth in stocks (i.e., less population and capital) and less depletion of resources). The simulation result is relatively robust to changes in the other two wage function parameters, ϕ_o and ϕ_s.

One takeaway from the results of the Fossil scenario of Table 3 and Figure 2 is that the overall dynamics are indistinguishable from the Renewable-Low(a) scenario. Thus, if only observing the outcomes of the HARMONEY model, one could not tell if the simulation assumed a natural resource as a fossil or regenerative renewable resource. This could help explain why people can disagree on whether or not it matters if the economy depends on finite natural resources. In context of the quote from Weitzman within the first section of this paper, to a "contemporary economist" a collapsing economy might manifest itself as a lack of investment. Higher investment avoids full and rapid collapse. However, to an "average ecologist" or biophysical economist a collapsing economy might manifest itself as a lack of resources. In other words, the present model shows that without knowledge of the inner-workings of an economy, we might not be able to distinguish an economy that depends on fossil resources from one that depends on renewable resources but has insufficient capital investment. The case study that compares model results to U.S. data expands this discussion of how we can interpret the HARMONEY model in the context of extracting previously inaccessible fossil resources.

3.3 Comparison of Constant and Variable Markup Simulations

The variable markup simulations show many similarities while also having some distinctly different dynamics than the constant markup scenarios. Here we highlight a couple of key features. See Supplemental Section SI.7 and Figure S.5 for a fuller description and display of variable markup results.

As in the case of the constant markup scenarios, the variable markup scenarios can exhibit a full rapid collapse, similar to the Renewable-Low scenarios, when investment increases mainly as a multiple of depreciation ($\kappa_0 > 1.0, \kappa_1 = 1.0$). The variable markup scenarios can also exhibit a slower "non-collapsing" dynamic with steadily increasing depreciation share and decreasing wage share, similar to the Renewable-High scenarios, when investment increases mainly as a multiple of profit ($\kappa_0 = 1.0, \kappa_1 > 1.0$). However, a key difference is that the variable markup dynamics are much slower because investment is lower by an order of magnitude (Figure S.5(r)). Lower investment leads to less capital and higher peak population. Investment is lower due to lower profits. Profits are lower due to lower cost markups that decline to values below 0.01 from initial values comparable to those used in the constant markup case (Figure S.5(v) and (w)).

In the variable markup simulations that do not end in rapid collapse, a second key difference from the constant markup case is that after debt ratio rises and declines, it stabilizes toward zero. The feedbacks that lower profit rate, and thus the variable cost markups, act to stabilize investment and debt ratios once the system reaches maximum participation rate. Thus, a low markup is consistent with a steady state scenario with full employment and no debt as a long-term outcome of some variable markup scenarios. However, it is important to keep in mind that the variable markup value does not decline to zero, and wage share continues to decline (Figure S.5(j)) in the long run with increasing capital (Figure S.5(c)) and depreciation share (Figure S.5(z)). Thus, the system is not at a complete steady state or equilibrium.

4 Case Study: Comparing Model and U.S. Trends

We now compare the dynamic evolution of structural changes in the model to those of the United States (U.S.) over the last 70–90 years. Specifically we compare results from the Renewable-High investment scenario using constant cost markup. While the model trends show important similarity to those of the U.S., we caution that the model is not calibrated to the U.S. or any economy. Despite the exclusion of many important economic features, the comparison to U.S. data indicates that the model characterizes important underlying processes that govern long-term growth and structural change in an economy such that of the U.S. For our model-U.S. comparison, the general sequence of long-trends and structural change are important, not the relation of magnitudes of variables or specific model times to specific years in the U.S. data.

Three reasons support comparison of the model to the U.S. First, the U.S. is relatively resource self-sufficient, and the model assumes full self-sufficiency. Second, our investment behavior matches that of the U.S. in that gross investment is significantly greater than net profits. Third, both the model and U.S. data exhibit an initial period with increasing per capita resource, or energy, consumption followed by one with approximately constant per capita consumption.

This third reason is critical. The HARMONEY model assumption of an economy extracting a regenerative renewable resource inherently simulates a trend of increasing and then steady per capita resource extraction (i.e., as long as model parameters are not set to not trigger the full rapid collapse of the Renewable-Low scenarios). Thus, our model is useful for answering the question "How might the economy respond when transitioning from a period of increasing per capita resources consumption to one with steady per capita resources consumption?" It just so happens that the U.S. economy also exhibits this trend for energy consumption. This is despite the fact that U.S. economic growth over the last two centuries, and that of all modern economies generally, has been overwhelmingly powered by consuming fossil fuels. In this context, it is important to keep in mind that one could interpret our renewable scenarios as fossil extraction scenarios in which society continually finds new extractable fossil resources that were previously unknown to exist or technically impossible to extract. The regeneration rate, γ, would then represent a rate of discovery. This concept is akin to converting a mineral *resource* into a mineral *reserve*, where reserves are technically and economically accessible but resources are not.

4.1 Wage Share

An important structural metric of an economy is the proportion of GDP that accrues to workers versus owners of capital as profits and rents. The share of U.S. GDP allocated to wages (wage share) and total employee compensation (compensation share) both began to decline after peak per capita energy consumption in the 1970s. With our model resource as analogous to U.S. energy, our model exhibits this same relationship. Model wage share and household income share (wages plus bank dividends) are approximately constant before transitioning to a period when they steadily decline starting soon after the peak in per capita resource extraction. Figure 4 compares the Renewable-High(a) constant markup scenario result to U.S. data. The model results qualitatively mimic the pattern in U.S. wage share (Figure 4(b)).

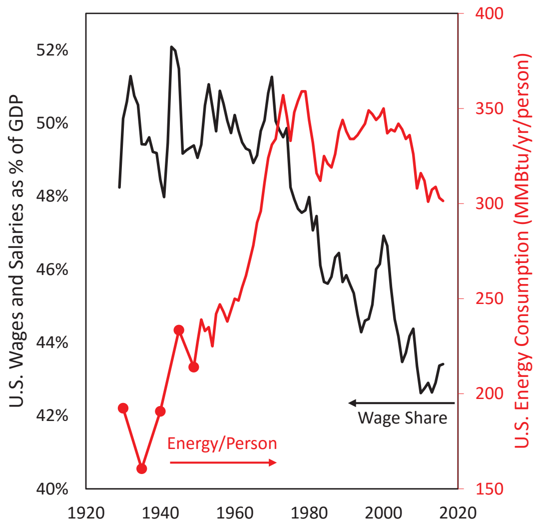
From the 1930s until the early 1970s, the share of U.S. GDP allocated to salaries and wages was approximately constant at 50% ±2%. Since the early 1970s it declined to 42-43% in the 2010s. U.S. employee compensation adds health and retirement benefits to wages, and its share also began to decline in the early 1970s after rising during the previous forty years due to increasing employee benefits related to efforts of the New Deal [58]. The early 1970s also mark a breakpoint in trend for U.S. per capita energy consumption. From the 1930s until 1973, per capita energy consumption rose 3.2%/year from approximately 160 MMBtu/yr/person to just over 350 MMBtu/yr/person. After 1973 per capita energy consumption remained relatively constant before declining after the year 2000.

Importantly, our model exhibits the same general long-term pattern in wage share as the post-Great Depression U.S. It does so without any explicit changes in assumptions relating to policy, labor bargaining power, or geopolitical interventions in the energy market. The post-1970s decline in U.S. wage share characterizes many Western economies, and many attribute this decline to a reduction in the "bargaining power" of labor. The two decades before the 1970s had higher union membership, and some union contracts in the U.S. stipulated that wages increase with inflation. Thus, lower wage share signals a shift away from labor power. Also, geopolitical actions influenced the U.S. and global economy in the 1970s. The Arab oil embargo occurred in late 1973, and the Organization of Petroleum Exporting Countries increased the posted price of oil from $5.12 to $11.65 per barrel at the beginning of 1974. The 1979 Iranian Revolution caused a reduction of 4 million barrels per day of production from the world market. This 1979 event triggered a second major oil price spike in the decade, further restricting, via physical shortage and economic demand destruction, per capita energy consumption in the U.S. Both the 1973/74 and 1979 events would have had lesser influence had the U.S. not reached a peak in domestic oil extraction in 1970.

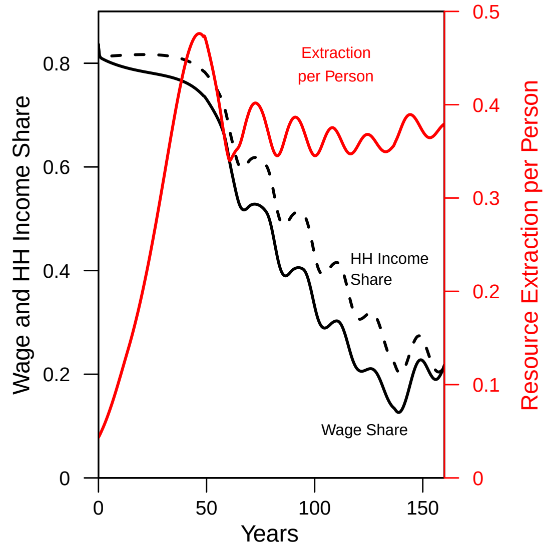
Regardless of the reasons for reaching peak per capita resource extraction or energy consumption, in both the U.S. data and our model, the total share of value added going to labor, or households more generally, remains relatively constant with increasing per capita resource extraction, and it decreases during constant per capita resource extraction. Wage share can decline in response to an increasing share in any of the other three components of value added (profit, depreciation, or interest payments). Our simulations demonstrate the rate of investment and cost markup can determine which component increases. For the constant markup scenarios, profit dominates for the Renewable-Low scenarios while depreciation dominates for the Renewable-High scenarios. For the variable markup scenarios that do not exhibit full collapse, interest payments dominate the dynamics before the model reaches maximum participation rate and debt ratio. After reaching maximum participation rate, depreciation share slowly dominates. In the long run, wage share declines because the capital stock and depreciation continuously increase.

4.2 Debt Ratio

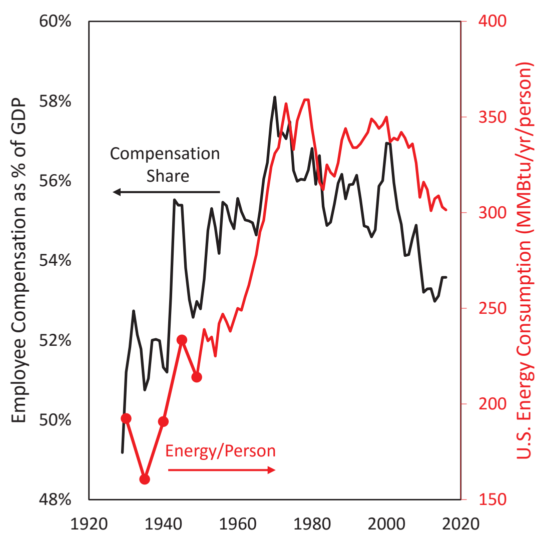
The U.S. data and model also exhibit similar changes in trend for debt ratio. Figure 5(a) shows that U.S. corporate and financial debt ratio grew slowly before the 1970s but much more rapidly after. In the same way, the model debt ratio grows slowly before peak per capita resource extraction rates and much more rapidly immediately after. Importantly, the modeled post-peak extraction increase in debt ratio does not necessarily occur due to continuously increasing investment rates. Even when total real net investment remains approximately constant, debt accumulates faster than net output (see Renewable-High(a) scenario with $\kappa_{i,0} = 1.0$, $\kappa_{i,1} = 1.5$, Figures 2(r)



(a) U.S. data

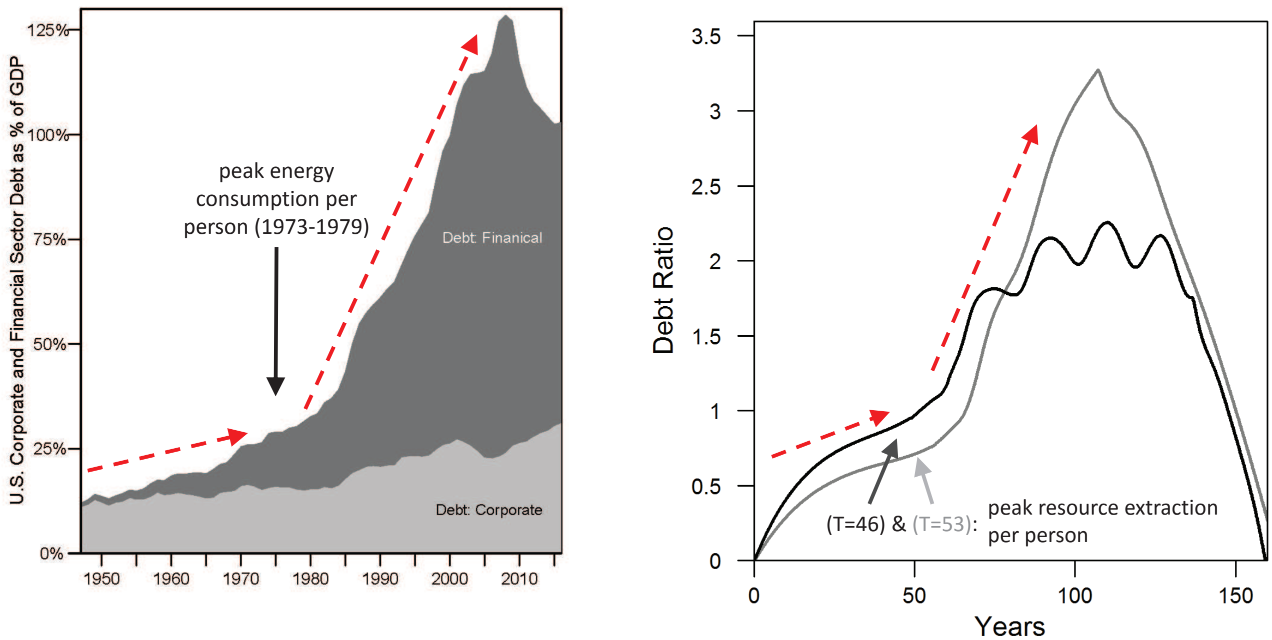


(b) Renewable-High(a) Scenario.



(c) U.S. data

Figure 4: The modeled wage dynamics before and after a peak in per capita resource extraction mimic those of the U.S. before and after peak per capita energy consumption occurred in the 1970s. (a) The U.S. data for wage share (wages and salaries divided by GDP) and primary energy consumption per person. (b) Model result from Renewable-High Investment(a) scenario (see Figure 2 and Table 3). (c) The U.S. data for compensation share (wages, salaries, and benefits divided by GDP) and primary energy consumption per person. U.S. wage data are from the Bureau of Economic Analysis "Table 1.12. National Income by Type of Income". U.S. energy data are from the Energy Information Administration Monthly Energy Review Table 1.1 and Appendix D1.



(a) U.S. Data

(b) Model results for Renewable-High(a) (black line) and Renewable-High(b) (gray line) scenarios.

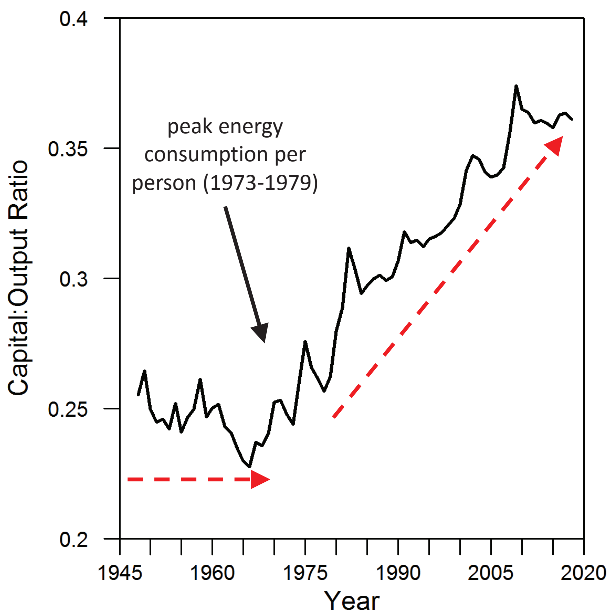
Figure 5: Both the (a) U.S. data and (b) model (constant cost markup with high investment) show a slow rise in private debt ratio before a more rapid increase. The transition occurs soon after the peak in per capita energy consumption for the U.S. and peak in resource extraction per person for the model. U.S. data are from U.S. Federal Reserve Bank z.1 Financial Accounts of the United States, Table L.208 (Debt, listed as liabilities by sector).

and SI.6(r)).

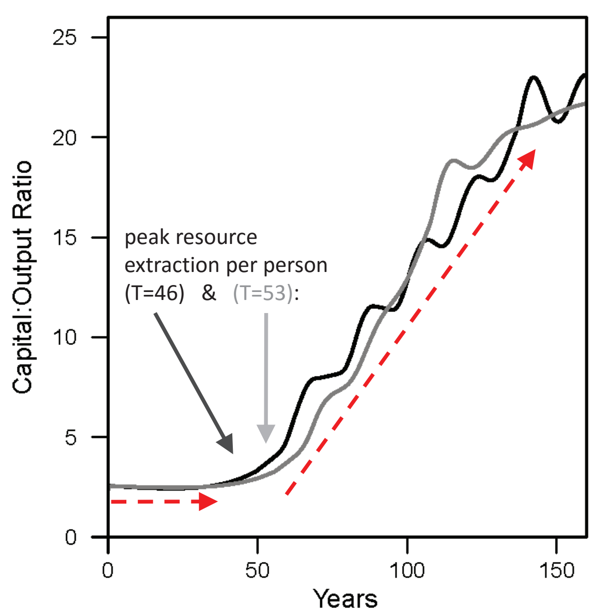
4.3 Capital:Output Ratio

A 2015 Chicago Fed Letter by Gourio and Klier notes the 40-year increase in U.S. capital output ratio, starting in the mid-1970s [59]. We replicate those data in Figure 6(a). The data are from the U.S. Bureau of Labor Statistics (BLS) multifactor productivity program. As noted in Gourio and Klier (2015), the measure of capital from "The BLS index is a better measure of aggregate capital as an input in production [than the capital measure from of the U.S. Bureau of Economic Analysis fixed-assets tables] because it aggregates capital stocks using estimated rental prices." This is a good comparison for the physically-productive capital within our model. Further, their measure of economic output is "nonfarm business sector output". Figure 6(b) shows the comparable capital:output ratio from Renewable-High scenarios (a) and (b). Here we calculate capital as the real monetary value of capital equal to $P_{g,real}(K_g + K_e)$. Supplemental Section SI.1 shows indices for calculating real prices and monetary output. We use net output as total real net output equal to $Y_{g,real} + Y_{e,real}$. Since the real price of goods varies less than 25%, the same capital:output trend occurs if we plot the physical quantity of capital relative to net output.

Both the U.S. data and the Renewable-High scenario results exhibit the same trends. Just as in the case of debt ratio and wage share, there is a distinct transition about the time of peak energy consumption per person in the U.S. Similarly, for the model results, the transition occurs about the time of peak per capita resource



(a) U.S. Data



(b) Model results for Renewable-High(a) (black line) and Renewable-High(b) (gray line) scenarios.

Figure 6: Both the (a) U.S. data (1948-2018) and (b) model (constant cost markup with high investment) show a period of constant capital:output ratio before a period of increasin capital:output ratio. The transition occurs soon after the peak in per capita energy consumption for the U.S. and peak in resource extraction per person for the model. U.S. data are from U.S. Bureau of Labor Statistics multifactor productivity program as specified in [59].

extraction. The authors of the Chicago Fed Letter attribute the post-1970s increase in capital:output ratio to "...the accumulation of assets thanks to higher productivity—in particular the availability of cheaper, more efficient capital goods, such as computers." However, our model shows how this trend can represent a decline in capital productivity and a constraint on resource extraction. This is because in our model extraction capital requires more resource input to extract a depleting resource, and thus its capital productivity, $\delta_y y$ declines with a smaller resource base. Thus, it seems more intuitive to think that the increase in capital:output ratio is due to maintaining relatively high investment when net output is restricted to a slow increase constrained by resource extraction. This dynamic implies less productive capital, not more.

5 Discussion

It is the combination of factors and feedbacks within the model, not individual components, that provide insights into long-term economic macrodynamics. Major insights from this model stem first from the comparison of economic trends before and after reaching a peak in resource extraction and second from comparing the model to U.S. data.

While the present analysis excludes explicit economic policy or changes to implied policies (e.g., via changes to parameters in the wage function), the takeaway from our model results is not that policy interventions are unimportant. Policies that influence investment, pricing (e.g., cost markups), and collective bargaining can affect wages. The takeaway from the model is that economic outcomes are also dependent on resource considerations. One set of policies and rules will not produce the same socioeconomic outcomes in times of increasing, constant, or decreasing resources consumption. Our model shows that economies change structure even assuming constant technology and rules that govern investment and wages. While U.S. labor unions successfully bargained for higher wages during the three post-World War II decades of increasing per capita resource consumption, they have not been as successful since the energy constraints of the 1970s. For example, in 1981 U.S. president Ronald Reagan famously fired the vast majority of unionized air traffic controllers that went on strike. This sent a clear signal to both labor and capital owners that unions had lost significant bargaining power. In the model, the interplay of population, resource extraction limits, and investment drive the major distributional changes within value added. These combined factors are necessary to provide the rationale for the slow decline in wage share after reaching peak resource consumption in the model, and we infer the same mechanism for the post-1970s U.S. that experienced stagnant per capita energy consumption.

Were the U.S. labor policies under Reagan a *cause* of wage share decline or an *inevitable effect* from stagnating resource consumption? Why was U.S. labor unable to continue bargaining for the same wage and compensation shares it experienced in the early 1970s? Why did the U.S. private debt ratio and capital:output ratio both increase rapidly after the 1970s? While the scope of these questions is beyond that of this paper, our model encompasses a coupled biophysical-economic mechanism to explain how these important systemic changes in economic metrics can be triggered by stagnating resource consumption, as observed in the U.S. postwar economy, independent of explicit or exogenous political factors. We cannot separate economic and political decisions from the resource context within which they are made.

If any of the other three components of value added (interest, depreciation, or net profits) increase substantially, then the model structure dictates that wage share is the factor that declines. We do not model bankruptcy that could rapidly lower the value of capital, and thus money flows to depreciation. We do not model loan defaults that could reduce debt levels, and thus interest payments. In this sense, the model structure prioritizes depreciation and interest payments. They must be paid. The profit share can also increase. However, it does so to a significant degree only in the Fossil and Renewable-Low investment scenarios that end in full and rapid system collapse.

Thus, the Renewable-High scenarios provide the most insight into contemporary economic trends. Perhaps counterintuitively, these scenarios indicate that higher investment in capital both extracts resources more quickly and prevents a full rapid collapse because population grows less. Aside from the comparisons already made in the U.S. case study, an additional parallel to the Renewable-High investment scenario is the extremely rapid investment during World War II. Robert Gordon notes that "The number of machine tools in the U.S. doubled from 1940 to 1945, and almost all of these new machine tools were paid for by the government rather than by private firms."[58][2] This high investment helped dramatically increase the employment rate coming out of the Great Depression. Similarly, when resources are abundant in the model, higher investment also leads to a more rapid increase in employment rate.

Crucially, the underlying economic processes within the HARMONEY model govern trade-offs we must make in reaching sustainability goals. If constant resource consumption and profit share translate to declining wages in a capitalist economy, this poses difficulties for simultaneously limiting material consumption while increasing or maintaining high income equality. In addition, the model results are consistent with research positing long-term cycles of societal growth and development [60] and linkages between economic complexity, or structure, and resources consumption and cost [44, 45, 49].

6 Conclusion

The macroeconomic framework developed in this paper endogenously couples biophysical and economic variables by consistently tracking both stocks and flows of population, resources, and money. Crucial model components are the specification of two sectors, resource extraction and capital goods production, along with their required intermediate consumption of resources to both operate capital within each sector (i.e., fuel) and become embodied in capital (i.e., capital is made of resources). As the natural resource depletes, a larger fraction of resources is needed as input for extraction, thus embedding the concept of net energy and linking it to prices. The model thus enables investigation into how the size and structure of the economy evolve to changes in both the rate of investment and the choice of a markup that determines prices as a multiple of sector costs. We explore impacts of forming prices based on a constant markup versus one that varies in proportion to profit rates that decline over time but stay above zero.

We find that both cost markup assumptions can lead to a rapid system collapse or a near steady state trend. The trend is "near steady state" because while resources and population stay at constant levels, capital continues to accumulate, thus driving up the share of value added allocated to depreciation while driving down the share allocated to wages. Using the variable cost markup, rapid collapse occurs when scaling investment as a multiple of depreciation, but the near steady state trend occurs with "very low" investment scaled as a multiple of profit. Prices based on the constant cost markup, at an order of magnitude larger than the long-term variable markup, drive higher profits and investment that in turn accelerate the model dynamics. However, if investment is too low using a constant markup, the system reaches a full rapid collapse because net resource extraction eventually falls below the specified minimum threshold household resource consumption. If investment is high enough, capital grows so fast that peak extraction occurs before population grows too large for the system to collapse. Perhaps counterintuitively, both "very low" and "high enough" investment rates avoid a systemic full and rapid collapse, while moderate investment does not.

The high investment scenario with a constant cost markup compares best to trends of the U.S. data over the last ninety years. As such, this scenario model results provide a theoretical explanation linking the U.S. peak in per capita energy consumption in the early 1970s to the subsequent five decade-long decline in wage share. The explanation centers on understanding the size of the population and capital stock at the time of reaching a peak in resource extraction. With this theoretical and structural foundation, the model framework serves as a base from which to add features and calibrate to real-world economies to explore future energy-economic scenarios, such as a low-carbon energy transition.

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[2] p. 553, [58]

8 Open Source Model Code

The model is programmed in the R language. It is meant to be used as an open-source code. The two codes used to produce the results in this paper are posted alongside the link to the paper on the publications page of the author's website (`careyking.com`). One code assumes pricing via the constant markup on costs, and another code assumes pricing via the variable markup on costs.

9 Competing Interests

The author declares no competing interests and knows of no organizations that may gain or lose financially through this publication. During the performance of this research, the author was declared to have no financial conflicts of interest via his employer, The University of Texas at Austin.

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A.1 Nomenclature

Table A.1: Definition and nomenclature of symbols used in the HARMONEY model equations.

Symbol	Description	Units
a_e	Labor productivity of resource extraction sector	resources/person/time
a_{ee}	Resources input required per unit of gross nature extraction	(resource/time)/(resource/time)
a_{eg}	Resources input required per unit of gross goods production	(resource/time)/(good/time)
a_g	Labor productivity of goods sector	goods/person/time
a_{ge}	Goods input required per unit of gross resource extraction	(good/time)/(resource/time)
a_{gg}	Goods input required per unit of gross goods production	(good/time)/(good/time)
c_e	Unit cost of production of extraction sector	money/resource
c_g	Unit cost of production of goods sector	money/good
C_e	Consumption of extraction sector output by households	money/time
C_g	Consumption of goods sector output by households	money/time
CU_e, CU_g	Capacity utilization of extraction and goods capital	–
D	Total private debt (of firms)	money
D_e, D_g	Debts of extraction and goods sectors	money
g	Physical inventory of goods	goods
$GEPR$	Gross external power ratio	(resource/time)/(resource/time)
I	Total private investment (by firms)	money/time
I_e, I_g	Investment by extraction and goods sector (by firms)	money/time
I_e^g, I_g^g	Investment by extraction and goods sector (by firms)	goods/time
IC_e, IC_g	Inventory coverage of extraction and goods capital	–
$\text{INV}_e, \text{INV}_g$	Value of inventory of extraction and goods sector	money
$\Delta\text{INV}_e, \Delta\text{INV}_g$	Change in value of inventory of extraction and goods sector	money/time
K	Total capital	goods
K_e, K_g	Capital for resources extraction and goods production	goods
L	Total labor	person
L_e, L_g	Labor for resources extraction and goods production	person
M	Total deposits (of money) in banks	money
M^h	Household deposits (of money)	money
N	Population	person
$NEPR$	Net external power ratio	(resource/time)/(resource/time)
$\text{NPR}_{economy}$	Net power ratio	(resource/time)/(resource/time)
P_e	Price of extraction output	money/resource
P_g	Price of goods output	money/good
r_L, r_M	Interest rate (L: on loans; M: for household deposits)	1/time
s	Threshold household consumption of resources per person	resource/person/time
S^b	Saving of banks	money/time
S^h	Saving of households	money/time
V	Total value added of the economy	money/time
V_e, V_g	Value added of extraction and goods sectors	money/time
w_H	Physical inventory of resources, wealth (per [1])	resource
w	Wage per person	money/person/time
W_e, W_g	Wages for labor in extraction and goods sector ($W = wL$)	money/time

Table A.1: (continued)

Symbol	Description	Units
X_e	Gross output of extraction sector (= resources extraction)	resource/time
X_g	Gross output of goods sector (= goods production)	good/time
X^b	Net worth of banks	money
X^h	Net worth of households	money
X_e^f, X_g^f	Net worth of extraction and goods firms	money
X^{tot}	Net worth of entire economy	money
y	Resources in the environment	resource
y_{X_g}	Resource input required per unit of goods sector output	goods/ (resource/time)
Y	Total net economic output of the economy	money/time
Y_e, Y_g	Net economic output of extraction and goods sectors	money/time
α_N	Death rate of population	1/time
β_N	Birth rate of population	1/time
γ	Regeneration rate of resources	1/time
δ	Depreciation rate of capital	1/time
δ_y	Technology parameter affecting resources extraction rate	1/(time $\times$ good)
η_e, η_g	resources input to operate extraction and goods capital	resource/(time$\times$good)
κ_0	Investment function parameter (equation 6)	–
κ_1	Investment function parameter (equation 6)	–
λ_y	Maximum size of natural resources with no depletion	resource
λ_N	Participation (employment) rate	–
$\lambda_{N,o}$	Wage function parameter (Equation S.1), equilibrium employment	–
μ_e, μ_g	Cost markup for extraction and goods sector (in price Equation 12)	–
ν_e	Capital to output ratio for extraction output	goods/ (resource/time)
ν_g	Capital to output ratio for goods output	goods/ (goods/time)
Π_b	Bank dividends	money/time
Π_e, Π_g	Net profit of extraction and goods sectors	money/time
π_e, π_g	Profit share of extraction and goods sectors	–
$\pi_{r,e}, \pi_{r,g}$	Profit rate of extraction and goods sectors	1/time
ρ_e	Minimum per capita household consumption of extraction sector output	resources/(person/time)
ρ_g	Minimum per capita household consumption of goods sector output	goods/(person/time)
$\tau_{CU,e}, \tau_{CU,g}$	Time delay for extraction and goods CU differential (lag) equation	time
$\tau_{IC,e}, \tau_{IC,g}$	Time delay for extraction and goods IC differential (lag) equation	time
$\tau_{P,e}, \tau_{P,g}$	Time delay for extraction and goods price differential (lag) equation	time
$\tau_{V,e}, \tau_{V,g}$	Time delay for extraction and goods value added differential (lag) equation	time
$\tau_{Y,e}, \tau_{Y,g}$	Time delay for extraction and goods net output differential (lag) equation	time
$\tau_{\Pi,e}, \tau_{\Pi,g}$	Time delay for extraction and goods profit differential (lag) equation	time
ϕ_{min}	Wage function parameter (Equation S.1)	1/time
ϕ_o	Wage function parameter (Equation S.1)	1/time
ϕ_s	Wage function parameter (Equation S.1)	1/time
ω	Wage share (of value added)	–

A.2 Parameter values used in for simulations

A.3 Setting initial conditions

In this appendix, we describe our procedure for setting a consistent set of initial conditions, and we follow this procedure for the results shown in this paper. The initial condition values for our base model run are given in Table A.3.

We first set the following initial states independently of other initial states: population (N_o), quantity of available resources (y_o), perceived inventory coverage of goods ($IC_{g,perceived,o}$) and extraction ($IC_{e,perceived,o}$) sectors, debt of goods and extraction sectors ($D_{g,o}$, $D_{e,o}$), and capital of goods and extraction sectors ($K_{g,o}$, $K_{e,o}$). Our base simulation also assumes initial available resources (y_o) as equal to the maximum available resources (λ_y), and labor productivity is constant and equal for both sectors ($a_g = a_e = 1$).

Using the initial inventory coverage we set the initial capacity utilization for each sector ($CU_{g,o}$, $CU_{e,o}$). Given initial capital and utilization, we calculate initial gross output of each sector ($X_{g,o}$, $X_{e,o}$). Using initial gross output, combined with initial labor productivity, we solve for initial labor for each sector ($L_{i,o} = X_{g,o}/a_{g,o}$). One must check that the initial labor is not so large as to be above the maximum participation rate. One can raise the initial labor productivity to lower the initial participation rate. Initial wage per person (w_o) is decided to then calculate total initial wages, $= w_o(L_{g,o} + L_{e,o})$.

Given the set initial interest rates (r_L and r_M), which can be arbitrary, we have all inputs to calculate the initial prices using Equation 16 when assuming a constant markup on price. When assuming the variable markup on price, to solve for initial prices using Equation 24, we must first assume an initial profit rate for each sector ($\pi_{r,g,o}$, $\pi_{r,e,o}$).

With initial prices now solved, we solve for the intermediate monetary transactions of the economy. We then solve for initial cost of goods and extracted resources ($c_{g,o}$, $c_{e,o}$). For the constant markup assumption, initial profit of each sector is solved as the fixed cost markup multiplied by the initial cost and initial output ($\Pi_{i,o} = \mu_{i,o} c_{i,o} X_{i,o}$). For the variable markup assumption, we first solve for the initial markup using Equation 19 before solving for initial sector profits.

Once profit is known, we solve for initial value added ($V_{i,o}$) and profit share ($\Pi_{i,o}/V_{i,o}$) for each sector. Initial investment ($I_{i,o}$) can now be solved via Equation 6. With known gross output, investment, and intermediate expenditures, we solve for initial net output ($Y_{i,o}$) using Equation 27. Initial household consumption is now solved via Equations 30 and 31.

To solve for the initial quantity of physical inventory of goods (g_o) and wealth ($w_{H,o}$, stored resources), we assume the initial targeted physical consumption of each sector is equal to initial total gross output. Then, $w_{H,o} = X_{e,o}/(1/\tau_{CU,e})$ and $g_o = X_{g,o}/(1/\tau_{CU,g})$. The initial value of inventory of each sector is solved as initial cost times the quantity of inventory ($INV_{i,e} = c_{e,o} w_{H,o}$, $INV_{i,g} = c_{g,o} g_o$).

A.3.1 Initial conditions for base run of model

Table A.2: Parameter values used in the base run of the HARMONEY model.

Symbol	Description	Value
a_{ge}	goods input required per unit of gross resources extraction	0.2
a_{gg}	goods input required per unit of gross goods production	0.1
$IC_{ref,g}$	reference (target) inventory coverage for goods sector	1
$IC_{ref,e}$	reference (target) inventory coverage for extraction sector	1
r_L	Interest rate (L: on loans)	0.05
r_M	Interest rate (M: for household deposits)	0.0
s	threshold household consumption of resources per person (Equation S.9)	0.08
y_{X_g}	resource input required per unit of goods sector output	0.1
α_m	minimum death rate of population (Equation S.9)	0.01
α_M	maximum death rate of population (Equation S.9)	0.07
β_N	Birth rate of population	0.03
δ_y	Resources extraction factor (constant markup renewable scenarios)	0.012
δ_y	Resources extraction factor (constant markup fossil scenario)	0.0012
δ_y	Resources extraction factor (variable markup renewable scenarios)	0.019
δ_y	Resources extraction factor (variable markup fossil scenario)	0.0019
γ	Regeneration rate of resources (renewable scenarios)	0.01
γ	Regeneration rate of resources (fossil scenarios)	0.0
δ	Depreciation rate of capital	0.03
η_e, η_g	resources input to operate extraction and goods capital	0.16
κ_0	Investment function parameter (Equation 6)	varies (1.0 to 1.4)
κ_1	Investment function parameter (Equation 6)	varies (1.0 to 1.55)
λ_y	Maximum size natural resources stock (renewable scenarios)	100
λ_y	Maximum size natural resources stock (fossil scenarios)	1000
$\lambda_{N,o}$	wage function parameter (Equation S.1), equilibrium participation rate	0.6
ν_g	Capital to output ratio for goods sector	1.5
ρ_e	Minimum household consumption of extraction sector output	0.01
ρ_g	Minimum household consumption of goods sector output	0
$\tau_{CU,e}$	time delay for extraction capacity utilization differential (lag) equation	0.3
$\tau_{CU,g}$	time delay for goods capacity utilization differential (lag) equation	0.3
$\tau_{IC,e}$	time delay for extraction inventory coverage differential (lag) equation	0.3
$\tau_{IC,g}$	time delay for goods inventory coverage differential (lag) equation	0.3
$\tau_{P,e}$	time delay for extraction price differential (lag) equation	0.3
$\tau_{P,g}$	time delay for goods price differential (lag) equation	0.3
$\tau_{V,e}$	time delay for extraction value added differential (lag) equation	0.3
$\tau_{V,g}$	time delay for goods net value added differential (lag) equation	0.3
$\tau_{Y,e}$	time delay for extraction net output differential (lag) equation	0.3
$\tau_{Y,g}$	time delay for goods net output differential (lag) equation	0.3
$\tau_{\Pi,e}$	time delay for extraction profit differential (lag) equation	0.3
$\tau_{\Pi,g}$	time delay for goods profit differential (lag) equation	0.3
ϕ_{min}	wage function parameter (Equation S.1)	-0.05
ϕ_o	wage function parameter (Equation S.1)	0
ϕ_s	wage function parameter (Equation S.1)	0.2

Table A.3: Initial values for state equations as used in the base run of our merged biophysical and economic model.

I. Initial conditions for core differential equations		
Symbol	Description	Value
a_o	Labor productivity (constant markup cases)	1
$D_{e,o}$	Debt for extraction sector	0.0
$D_{g,o}$	Debt for goods sector	0.0
g_o	Goods	0.37
$K_{e,o}$	Initial extraction capital	1.28
$K_{g,o}$	Initial goods capital	2.2
w_o	Wage per capita	1
$w_{H,o}$	Wealth	0.39
x_o	Population	30
y_o	Available resources	100
II. Initial conditions for (first order) lag differential equations		
$CU_{e,o}$	Capacity utilization for extraction sector	0.85
$CU_{g,o}$	Capacity utilization for goods sector	0.85
$IC_{e,perceived,o}$	Perceived inventory coverage for extraction sector	1
$IC_{g,perceived,o}$	Perceived inventory coverage for goods sector	1
	Constant Markup Simulations:	
$P_{e,o}$	Price of extraction sector output (constant markup)	1.77
$P_{g,o}$	Price of goods sector output (constant markup)	1.82
$V_{e,o}$	Value added for extraction sector (constant markup)	1.53
$V_{g,o}$	Value added for goods sector (constant markup)	1.54
$Y_{e,o}$	Net output from extraction sector (constant markup)	1.14
$Y_{g,o}$	Net output from goods sector (constant markup)	1.57
$\Pi_{e,o}$	Profit from extraction sector (constant markup)	0.151
$\Pi_{g,o}$	Profit from goods sector (constant markup)	0.172
	Variable Markup Simulations:	
$P_{e,o}$	Price of extraction sector output (variable markup)	1.59
$P_{g,o}$	Price of goods sector output (variable markup)	1.82
$V_{e,o}$	Value added for extraction sector (variable markup)	2.25
$V_{g,o}$	Value added for goods sector (variable markup)	1.57
$Y_{e,o}$	Net output from extraction sector (variable markup)	2.23
$Y_{g,o}$	Net output from goods sector (variable markup)	1.29
$\Pi_{e,o}$	Profit from extraction sector (variable markup)	0.116
$\Pi_{g,o}$	Profit from goods sector (variable markup)	0.200

SI.1 Additional Equations not otherwise Described in Methods

SI.1.1 Wage Function

For $\phi(\lambda_N)$ we use Keen's nonlinear exponential curve (equation S.1) that allows wages to rise increasingly rapidly at high participation but decrease slowly at low participation rate [28]. In Equation S.1 $\phi_{min} < 0$ is the minimum decline in wages at low participation rate, ϕ_o is the change in current wage defined at $\lambda_{N,o}$ (typically set $\phi_o = 0$ at an equilibrium participation rate $\lambda_{N,o}$), and ϕ_s defines the exponential rate of increase.

$$\phi(\lambda_N) = (\phi_o - \phi_{min})e^{\frac{\phi_s}{(\phi_o - \phi_{min})}(\lambda_N - \lambda_{N,o})} + \phi_{min} \tag{S.1}$$

SI.1.2 Price model 1 (additional): Constant Markup on Cost

A more full description of steps to solve for prices when assuming a constant markup on costs.

$$\begin{aligned}
c_g &= P_g a_{gg} + P_e a_{eg} + (wL_g + r_L D_g + \delta K_g P_g)/X_g \\
\frac{P_g}{1+\mu_g} &= P_g a_{gg} + P_e a_{eg} + (wL_g + r_L D_g + \delta K_g P_g)/X_g \\
\frac{wL_g + r_L D_g}{X_g} &= \left(\frac{1}{1+\mu_g} - a_{gg} - \frac{\delta K_g}{X_g}\right) P_g - (a_{eg})P_e
\end{aligned} \tag{S.2}$$

$$\begin{aligned}
c_e &= P_e a_{ee} + P_g a_{ge} + (wL_e + r_L D_e + \delta K_e P_g)/X_e \\
\frac{P_e}{1+\mu_e} &= P_e a_{ee} + P_g a_{ge} + (wL_e + r_L D_e + \delta K_e P_g)/X_e \\
\frac{wL_e + r_L D_e}{X_e} &= (-a_{ge} - \frac{\delta K_e}{X_e})P_g + \left(\frac{1}{1+\mu_e} - a_{ee}\right) P_e
\end{aligned} \tag{S.3}$$

Equations S.2 and S.3 combine in matrix form as:

$$\begin{aligned}
\begin{bmatrix} \frac{wL_g + r_L D_g}{X_g} \\ \frac{wL_e + r_L D_e}{X_e} \end{bmatrix} &= \begin{bmatrix} \frac{1}{1+\mu_g} & 0 \\ 0 & \frac{1}{1+\mu_e} \end{bmatrix} \begin{bmatrix} P_g \\ P_e \end{bmatrix} - \begin{bmatrix} a_{gg} & a_{eg} \\ a_{ge} & a_{ee} \end{bmatrix} \begin{bmatrix} P_g \\ P_e \end{bmatrix} - \begin{bmatrix} \frac{\delta K_g}{X_g} & 0 \\ \frac{\delta K_e}{X_e} & 0 \end{bmatrix} \begin{bmatrix} P_g \\ P_e \end{bmatrix} \\
\tilde{V} &= \hat{M}P - A^T P - \Delta P \\
\tilde{V} &= (\hat{M} - A^T - \Delta)P
\end{aligned} \tag{S.4}$$

SI.1.3 Price model 2 (additional): Variable Markup on Cost

A more full description of steps to solve for prices when assuming a variable markup on costs that adjusts based on profit rate.

$$P_i = \left(1 + \frac{\pi_{r,i} P_g K_i}{c_i X_i}\right) c_i \tag{S.5}$$

Using Equation S.5, we solve for prices for each sector:

$$\begin{aligned}
P_g &= P_g a_{gg} + P_e a_{eg} + (wL_g + r_L D_g + \delta K_g P_g)/X_g + \frac{\pi_{r,g} P_g K_g}{X_g} \\
\frac{wL_g + r_L D_g}{X_g} &= \left(1 - a_{gg} - \frac{\delta K_g}{X_g} - \frac{\pi_{r,g} K_g}{X_g}\right) P_g - (a_{eg})P_e
\end{aligned} \tag{S.6}$$

$$
\begin{aligned}
P_e &= P_e a_{ee} + P_g a_{ge} + (wL_e + r_L D_e + \delta K_e P_g)/X_e + \frac{\pi_{r,e} P_g K_e}{X_e} \\
\frac{wL_e + r_L D_e}{X_e} &= \left(-a_{ge} - \frac{\delta K_e}{X_e} - \frac{\pi_{r,e} K_e}{X_e}\right) P_g + (1 - a_{ee}) P_e
\end{aligned}
\tag{S.7}
$$

Equations S.6 and S.7 are combined in matrix form:

$$
\begin{aligned}
\begin{bmatrix} \frac{wL_g + r_L D_g}{X_g} \\ \frac{wL_e + r_L D_e}{X_e} \end{bmatrix} &= \begin{bmatrix} P_g \\ P_e \end{bmatrix} - \begin{bmatrix} a_{gg} & a_{eg} \\ a_{ge} & a_{ee} \end{bmatrix} \begin{bmatrix} P_g \\ P_e \end{bmatrix} - \begin{bmatrix} \frac{\delta K_g}{X_g} & 0 \\ \frac{\delta K_e}{X_e} & 0 \end{bmatrix} \begin{bmatrix} P_g \\ P_e \end{bmatrix} - \begin{bmatrix} \frac{\pi_{r,g} K_g}{X_g} & 0 \\ \frac{\pi_{r,e} K_e}{X_e} & 0 \end{bmatrix} \begin{bmatrix} P_g \\ P_e \end{bmatrix} \\
\tilde{V} &= P - A^T P - \Delta P - \Pi_r P \\
\tilde{V} &= (\mathbf{1} - A^T - \Delta - \Pi_r) P
\end{aligned}
\tag{S.8}
$$

SI.1.4 Death Rate

Equation S.9 describes the function for death rates, where s is the per capita resource consumption threshold below which death rates rise. As resource consumption declines below the threshold to zero the death rate linearly increases from a minimum value of α_m to a maximum "famine" death rate of α_M [1].

$$
\alpha_N \left(\frac{C_e}{P_e}\right) = \alpha_m + \max\left(0, \left[1 - \frac{\left(\frac{C_e}{P_e}\right)}{sN} (\alpha_M - \alpha_m)\right]\right)
\tag{S.9}
$$

SI.1.5 Inventory Coverage and Capacity Utilization

Perceived inventory coverage for each sector:

$$
\begin{aligned}
IC_{e,perceived} &= \frac{\frac{\text{perceived wealth, } w_H}{\text{time delay}}}{\text{targeted consumption of resources}} \\
IC_{e,perceived} &= \frac{\frac{w_H}{\tau_{IC,e}}}{C_e/P_e + a_{eg} X_g + a_{ee} X_e}
\end{aligned}
\tag{S.10}
$$

$$
\begin{aligned}
IC_{g,perceived} &= \frac{\frac{\text{perceived goods, } g}{\text{time delay}}}{\text{targeted consumption of goods}} \\
IC_{g,perceived} &= \frac{\frac{g}{\tau_{IC,g}}}{(C_g + I_e + I_g)/P_g + a_{gg} X_g + a_{ge} X_e}
\end{aligned}
\tag{S.11}
$$

Perceived capacity utilization of each sector, $0 \leq CU_{i,perceived} \leq 1$, is a lookup table that is an increasing function of the inverse of its respective inventory coverage [43]. When there is more inventory, capacity utilization decreases, and vice versa. The reference inventory coverage, $IC_{ref,i}$, is defined in the lookup table for capacity utilization as the amount of inventory present for capacity utilization to be at its reference value, $CU_{ref,i}$. We set $CU_{ref,i} = 0.85$ at $IC_{ref,i} = 1$.

$$
CU_{i,perceived} = f(IC_{i,perceived}^{-1})
\tag{S.12}
$$

The lookup table for CU of both sectors use input values as $IC_{i,perceived}^{-1} = [0, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 1e6]$ and output values as $CU_{i,perceived} = [0, 0.30, 0.55, 0.75, 0.85, 0.90, 0.94, 0.98, 0.99, 1, 1]$.

SI.1.6 Solving for Lagged Variables

For each sector i, the variables modeled using a first order lag are the capacity utilization (CU_i), perceived inventory coverage ($IC_{i,perceived}$), price (P_i), net output (Y_i), profit (Π_i), value added (V_i), and value of inventory (INV_i). Thus, each of $CU_i, IC_{i,perceived}, P_i, Y_i, \Pi_i, V_i$, and INV_i is modeled as an extra model state. In the model code, these lagged states are used to inform investment and all inputs needed to solve for a new "current" price. Additional model calculations use this newly calculated price, including calculations that updated the lagged states themselves. The states are updated via Equation S.13. For the inventory coverage of each sector (IC_i) it is the perceived inventory coverage ($IC_{i,percieved}$) that is modeled as a lagged state on the left hand side.

$$\dot{n}_{lagged} = (n - n_{lagged})/\tau \tag{S.13}$$

SI.1.7 Net Power Accounting

The derivation of the so-called "energy intensities" is shown here in more detail

$$\begin{bmatrix} 0 & 0 \\ \epsilon_{eg} & \epsilon_{ee} \end{bmatrix} = E = \hat{y}_{\text{extract}} \hat{X}^{-1} (\mathbf{1} - A)^{-1} \tag{S.14}$$

$$\begin{bmatrix} 0 & 0 \\ \epsilon_{eg} & \epsilon_{ee} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & X_e \end{bmatrix} \begin{bmatrix} X_g & 0 \\ 0 & X_e \end{bmatrix}^{-1} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} a_{gg} & a_{ge} \\ a_{eg} & a_{ee} \end{bmatrix} \right)^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} a_{gg} & a_{ge} \\ a_{eg} & a_{ee} \end{bmatrix} \right)^{-1}$$

The gross external power ratio (GEPR) is defined as:

$$\begin{aligned} GEPR &= \frac{\text{resource extraction}}{\text{resources required to invest in } K_e + \text{resources required to operate } K_e} \\ &= \frac{X_e}{\epsilon_{eg} \frac{I_e}{P_g} + a_{ee} X_e} \\ &= NEPR + 1 \end{aligned} \tag{S.15}$$

A more complete derivation of the economy-wide net power ratio (NPR):

$$\begin{aligned} NPR_{\text{economy}} &= \frac{\text{net resource output from economy}}{\text{resource extraction} - \text{net resource output from economy}} \\ &= \frac{1}{\frac{\text{gross power extracted}}{\text{net power output}} - 1} \\ &= \frac{1}{\epsilon_{ee} - 1} \end{aligned} \tag{S.16}$$

SI.1.8 Investment Behavior as a Function of Profit Rate

We could use an investment function that assumes monetary investment in each sector is an increasing smooth function of sector profit rate, $\kappa(\pi_r)$, multiplied by the value of capital of that sector, $P_g K_i$, as in [28]. Parameter $\kappa_{0,i}$ would then be some fraction of the value of capital invested independent of profit rate, and parameter $\kappa_{1,i}$ specifies how much investment is a multiple of profit rate (typically $\kappa_{1,i} \geq 1$). By selecting $\kappa_{0,i}$ to be equal to the depreciation rate, δ, one obtains the same level of investment as using Equation 6 of the manuscript.

$$\kappa_i(\pi_{r,i}) = max\{0, (\kappa_{0,i} + \kappa_{1,i}\pi_{r,i})\} \tag{S.17}$$

$$I_i = \kappa_i(\pi_{r,i}) P_g K_i = max\{0, (\kappa_{0,i} P_g K_i + \kappa_{1,i} \Pi_i)\} \tag{S.18}$$

SI.1.9 Inflation, Real Prices, and Real Interest Rate

We calculate inflation as a weighted change in the price of each sector output consumed by households using Equation S.19. The consumer price index (CPI) is as in Equation S.20. Real prices and wage are nominal values divided by CPI (e.g., $P_{g,real} = P_g/CPI$).

$$
\begin{aligned}
i = \dot{P} = & \frac{\Delta P_g(\frac{C_g}{P_g})}{P_g(\frac{C_g}{P_g})+P_e(\frac{C_e}{P_e})} + \frac{\Delta P_e(\frac{C_e}{P_e})}{P_g(\frac{C_g}{P_g})+P_e(\frac{C_e}{P_e})} \\
= & \frac{P_g(\frac{C_g}{P_g})}{P_g(\frac{C_g}{P_g})+P_e(\frac{C_e}{P_e})}\frac{\Delta P_g}{P_g} + \frac{P_e(\frac{C_e}{P_e})}{P_g(\frac{C_g}{P_g})+P_e(\frac{C_e}{P_e})}\frac{\Delta P_e}{P_e} \\
= & \frac{C_g}{C_g+C_e}\frac{\Delta P_g}{P_g} + \frac{C_e}{C_g+C_e}\frac{\Delta P_e}{P_e}
\end{aligned}
\tag{S.19}
$$

CPI based on inflation:

$$CPI = \prod_{t=1}^{T}(1+i_t) \tag{S.20}$$

The GDP deflator is nominal GDP divided by real GDP (where $Y_{i,t}$ is the net monetary output of sector i at time t). $P_{i,o}$ is the initial price of output from sector i. Real net output, value added, investment, debt, and consumption are calculated by dividing nominal values by the GDP deflator (e.g., $Y_{g,real} = Y_g/\text{GDP deflator}$).

$$
\begin{aligned}
\text{GDP deflator} &= \frac{P_{g,t}\left(\frac{Y_{g,t}}{P_{g,t}}\right) + P_{e,t}\left(\frac{Y_{e,t}}{P_{e,t}}\right)}{P_{g,o}\left(\frac{Y_{g,t}}{P_{g,t}}\right) + P_{e,o}\left(\frac{Y_{e,t}}{P_{e,t}}\right)} \\
&= \frac{Y_{g,t}+Y_{e,t}}{P_{g,o}\left(\frac{Y_{g,t}}{P_{g,t}}\right) + P_{e,o}\left(\frac{Y_{e,t}}{P_{e,t}}\right)}
\end{aligned}
\tag{S.21}
$$

Real interest rate is the nominal interest rate minus the rate of inflation.

$$r_{L,r} = \text{nominal interest rate} - \text{inflation} = r_L - i \tag{S.22}$$

SI.2 Stock-Flow Consistency: Balance Sheet, Transactions, and Flow of Funds Table

The model is stock-flow consistent in both money and physical units of resources and goods. Table S.1 shows the balance sheet, transactions, and flow of funds tables. The framework is similar to the Bank, Money, World model of Godley and Lavoie (2007) except we allow firms to have profit [4]. Household deposits, M^h, equal total firm debt. The net worth of firms is the value of their capital minus their debt. We model banks as having zero saving ($S^b = 0$) where bank net interest ($\Pi_b = r_L D - r_M M$) flows to households as bank dividends. Thus, we assume banks have zero net worth ($X^b = 0$). Table S.2 in the Supplementary material shows the monetary flow of the model in a Leontief input-output format.

The identity of "saving = investment" holds for this model. Household saving is equal to income minus consumption, $S^h = W + r_M M^h + \Pi_b - C$. Via the Financial Balances row of the transactions in Table S.1 (with $C = C_e + C_g$ as defined in Table S.1), we have Equation S.23 relating investment and saving from both sectors.

$$\begin{aligned}
S^h + \Pi_g + \Pi_e + P_g\delta(K_e + K_g) - I_g - I_e + S^b = 0 \\
S^h + (\Pi_g + \Pi_e) - (I_g + I_e - P_g\delta(K_e + K_g)) = 0 \text{ [since } S^b = 0] \\
S^h + (\Pi_g + \Pi_e) = (I_g + I_e) - P_g\delta(K_e + K_g) \\
\text{household saving} + \text{firm net saving (profit)} = \text{net investment}
\end{aligned} \tag{S.23}$$

Household saving, S^h, is equal to the change in household deposits, $\dot{M}^h$, which is equal to change in outstanding loans issued by banks to firms as debt ($S^h = \dot{M}^h = \dot{M} = \dot{D}$). Thus, since $S^h = \dot{D}$ (where D is debt of both the extraction and the goods sectors), the change in debt can be solved as in Equation S.24 where I, K, and Π are total firm gross investment, capital, and net profit with $I - P_g\delta K$ equal to net investment. This is the same as Equation 7.

$$\begin{aligned}
S^h + (\Pi_g + \Pi_e) + P_g\delta(K_e + K_g) - I_g - I_e + S^b &= 0 \\
(\dot{D}_e + \dot{D}_g) + (\Pi_g + \Pi_e) + P_g\delta(K_e + K_g) - (I_g + I_e) &= 0 \text{ [since } S^b = 0] \\
\dot{D} = (I - P_g\delta K) - \Pi &= \text{net investment - net profit}
\end{aligned} \tag{S.24}$$

Table S.1: Balance sheet, transactions, and flow of funds as formulated in the macroeconomic model where each of the sectors make individual investment decisions and profits

	Households	Extraction Firms		Goods Firms		Banks	Sum
Balance Sheet							
Capital		$P_g K_e$		$P_g K_g$			$P_g(K_g + K_e)$
Deposits	M^h					$-M$	
Debt (Loans)		$-D_e$		$-D_g$		D	
Sum (net worth)	X^h	X_e^f		X_g^f		$X^b = 0$	$X^{tot} = P_g(K_g + K_e)$ $= X^h + X^f$
Transactions		Current	Capital	Current	Capital		
Consumption	$-C$	C_e		C_g			
Investment			$-P_g I_g^e$	$P_g I_g^e + P_g I_g^g$	$-P_g I_g^g$		
Change in value of inventory		ΔINV_e	$-\Delta INV_e$	ΔINV_g	$-\Delta INV_g$		
Intermediate Sales		$+P_e a_{eg} X_g + P_e a_{ee} X_e$		$+P_g a_{ge} X_e + P_g a_{gg} X_g$			
Intermediate Purchases		$-P_g a_{ge} X_e - P_e a_{ee} X_e$		$-P_e a_{eg} X_g - P_g a_{gg} X_g$			
Accounting meme [Value Added]		$[V_e]$		$[V_g]$			
Wages	W	$-W_e$		$-W_g$			
Depreciation Allowance		$-P_g \delta K_e$	$P_g \delta K_e$	$-P_g \delta K_g$	$P_g \delta K_g$		
Interest on debt (loans)		$-r_L D_e$		$-r_L D_g$		$r_L D$	
Interest on deposits	$r_M M^h$					$-r_M M$	
Bank Dividends (net interest)	Π_b					$-\Pi_b$	
Financial Balances	S^h	Π_e	$P_g(\delta K_e - I_g^e) - \Delta INV_e$	Π_g	$P_g(\delta K_g - I_g^g) - \Delta INV_g$	S^b	
Flow of Funds							
Change in capital stock			$P_g \dot{K}_e$		$P_g \dot{K}_g$		$P_g(\dot{K}_e + \dot{K}_g)$
Gross Fixed Capital Formation			$P_g I_g^e$		$P_g I_g^g$		$P_g I$
Change in deposits	$\dot{M}^h$					$-\dot{M}$	
Change in debt (loans)			$-\dot{D}_e$		$-\dot{D}_g$	$\dot{D}$	
Column sum	S^h		Π_e		Π_g	S^b	$P_g I$
Change in net worth	$\dot{X}^h = S^h$		$\dot{X}_e^f = \Pi_e$		$\dot{X}_g^f = \Pi_g$	$\dot{X}^b = S^b = 0$	$\dot{X} = P_g I$

SI.3 Input-Output Format of Model

Table S.2: Input-Output matrix with value added and net output.

		Goods	Extraction	Net Output: Consumption	Net Output: Investment	Net Output: Change in Value of Inventory	Gross Output
	Goods	$P_g a_{gg} X_g$	$P_g a_{ge} X_e$	C_g	$P_g(I_g^g + I_e^g)$	$\Delta \mathrm{INV}_g$	$P_g X_g$
	Extraction	$P_e a_{eg} X_g$	$P_e a_{ee} X_e$	C_e	–	$\Delta \mathrm{INV}_e$	$P_e X_e$
Value Added	Profit	Π_g	Π_e				
Value Added	Wages	wL_g	wL_e				
Value Added	Interest Payments	$r_L D_g$	$r_L D_e$				
Value Added	Depreciation	$P_g \delta K_g$	$P_g \delta K_e$				
	Gross Output	$P_g X_g$	$P_e X_e$				

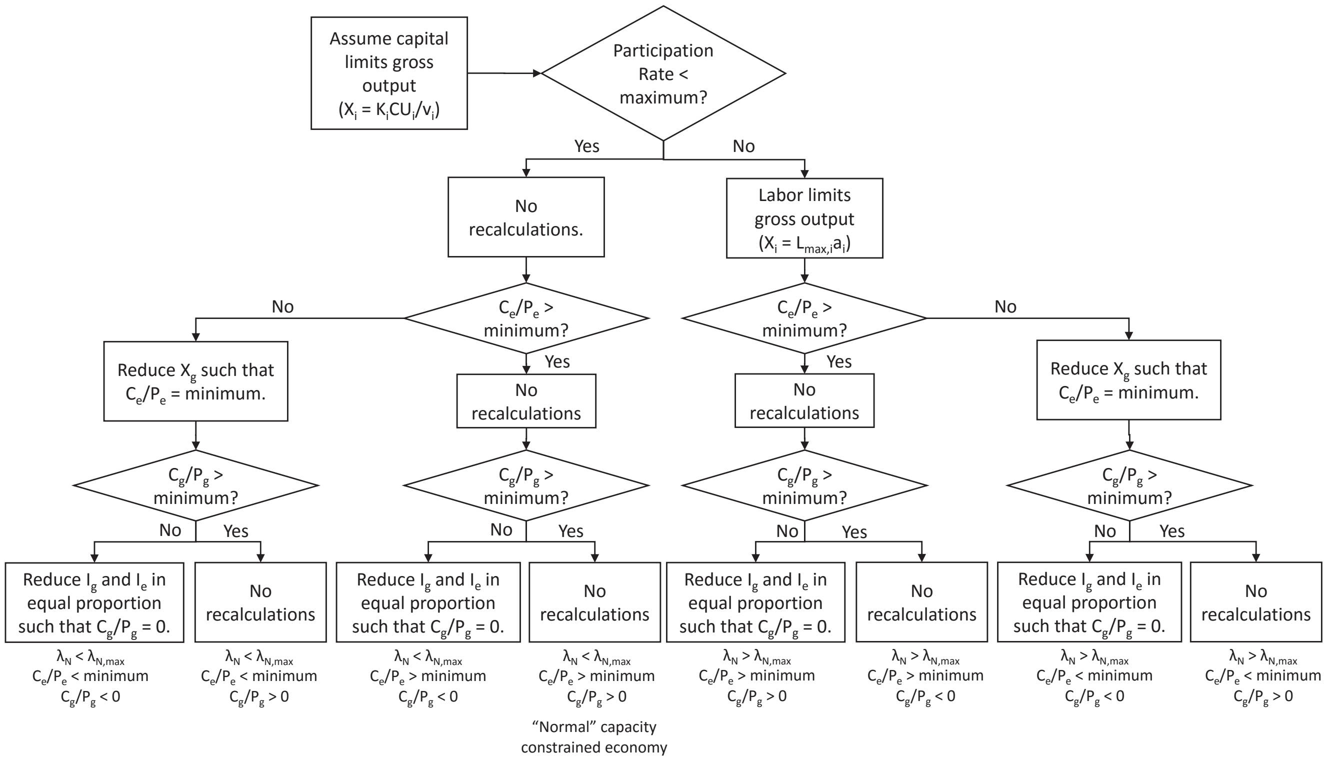

Figure S.2: The decision flow chart for downward adjustments of goods gross output, X_g, or investment, I_g and I_e, depending on whether the simulation approaches constraints in labor (necessary participation rate is higher than the assumed maximum level) or phsyical household consumption of resources and goods.

SI.4 Flow Chart of Algorithm within Model Code

Figure S.2 shows a decision tree that outlines how the model switches between different methods for solving for all system states. The reason this is necessary is because the model simulates periods when different factors must be constrained to realistic ranges.

We note that this algorithm is only one way to solve the model, and it derives from our model assumptions. For example, many post-Keynesian models assume that household consumption is a fraction of total household income, and then they calculate economic net output as investment plus consumption [4]. This avoids any issues in dealing with negative consumption. However, we assume that consumption is the residual net output after accounting for investment and change in value in inventories. This forces us to put constraints on the model to account for consumption that is too low.

We check for three conditions. Depending upon the result for each condition, we use a different order of solving for certain model variables.

The default assumption in the algorithm assumes all of the three below stated conditions are true. This effectively means that capital is constraining the output of the economy and there is enough household physical consumption of both goods and resources. Under this default assumption, gross output, labor, investment, and wages are calculated before checking the three conditions.

SI.4.1 Condition 1: Participation rate < maximum participation rate ($\lambda_N < \lambda_{N,\max}$)

The algorithm first calculates gross output based on the last known capital, capacity utilization, and capital-output ratio. It then solves for the necessary labor using the Leontief production function for each sector. If the calculated participation rate $\lambda_N < \lambda_{N,\max}$, then $X_i = \frac{K_i CU_i}{v_i}$ and the capacity utilization is updated using the lookup table as a function of inventory coverage.

Otherwise, if the calculated participation rate $\lambda_N > \lambda_{N,\max}$, then total labor is set at its maximum value, $L_{max} = \lambda_{N,\max} \times N$. We assume each sector lowers its quantity of labor by the same fraction, $L_{reduce} = (\lambda_N - \lambda_{N,\max})/\lambda_N$, such that updated labor quantities are $L_{i,updated} = L_i(1 - L_{reduce})$. Note that the total labor in each sector is not generally equal, or $L_{e,updated}$ does not generally equal $L_{g,updated}$.

Sector output now equals $X_i = L_{i,updated} a_i$. Capacity utilization is then solved from the Leontief condition such that $CU_i = \frac{v_i L_{i,max} a_i}{K_i}$. Total wages in each sector, $W_e = wL_e$ and $W_g = wL_g$, are updated based upon the updated labor quantities.

After solving for labor, the model solves for updated wages, prices, cost, value of inventory, profit, value added, net output, change in value of inventory, household consumption (monetary and physical), and change in physical inventory. The algorithm then moves to the next condition.

SI.4.2 Condition 2: Household resources consumption > minimum consumption ($C_e/P_e > \rho_e N$)

The minimum level of real household consumption must be greater than some nominal number greater than zero that is set by a minimum per capita resources consumption threshold, ρ_e. Thus, total minimum consumption is $(C_e/P_e)_{minimum} = \rho_e N$. The setting of a minimum resources consumption threshold above zero (1) serves to prevent having a death rate less than 100% at zero household resources consumption (e.g., if there was no food for consumption, everyone would die rather shortly), and (2) assumes that society would allocate a minimum amount of food to households at the expense of increased capital investment (i.e., that society would not choose to maximize capital investment while letting death rates reach high values due to starvation).

If $C_e/P_e > \rho_e N$, then no updates are made to calculated values that assume this condition is true.

If $C_e/P_e < \rho_e N$, we reduce X_g to reduce intermediate consumption of resources by the goods sector. The goods sector gross output required when household resources consumption is at its minimum value is solved using the equation for the physical flow of resources ($X_e = A_{ee}X_e + A_{eg}X_g + C_e/P_e + \Delta(w_H)$) where $\Delta(w_H)$ is the change in physical inventory of resources, or change in "wealth":

$$X_{g,updated} = \left(X_e(1 - A_{ee}) - \left(\frac{C_e}{P_e}\right)_{min} - \Delta(w_H) \right) / A_{eg} \tag{S.25}$$

Because the updated goods output, $X_{g,updated}$, is now lower than the previously assumed X_g, we reduce goods sector net output by the fraction reduction in X_g, which is equal to $X_{g,reduce} = \frac{X_g - X_{g,updated}}{X_g}$. Both household goods consumption and investment in each sector is reduced by the same fraction $X_{g,reduce}$. Equations S.26 and S.27 shows an adjustment to monetary goods consumption investment, and both physical goods consumption and physical investment can be calculated by dividing by the current price of goods, P_g.

$$C_{g,updated} = C_g(1 - X_{g,reduce}) \tag{S.26}$$

$$I_{i,updated} = I_i(1 - X_{g,reduce}) \tag{S.27}$$

Capacity utilization, labor, and wages for the goods sector are updated (reduced) based upon $X_{g,updated}$. At this point, all values needed to calculate prices have been updated, and the solution code solves for all other model variables before moving to check the next condition.

SI.4.3 Condition 3: Household goods consumption > 0 ($C_g/P_g > \rho_g$)

We set the minimum level of physical goods consumption at zero ($(C_g/P_g)_{minimum} = \rho_g N = 0$, or $\rho_g = 0$). Minimum consumption need only be greater than zero ($\rho_g > 0$) because negative physical consumption is not possible.

If $(C_g/P_g) \geq \rho_g$, then the model proceeds with the existing values for its variables. If $(C_g/P_g) < \rho_g$, then total investment is reduced by the amount $-(C_g)$, which is a positive number, and C_g is set to zero. Recall that household goods consumption C_g is in units of money per time, whereas (C_g/P_g) is in units of goods per time. Thus, since we cannot reduce investment greater than 100%, the fraction by which to reduce investment is as follows:

$$I_{\text{reduce fraction}} = \min \left\{ 1, \frac{-C_g}{I_e + I_g} \right\} \tag{S.28}$$

The updated investment in each sector is reduced by this same fraction as:

$$I_{i,updated} = I_i (1 - I_{\text{reduce fraction}}) \tag{S.29}$$

Now capacity utilization for the extraction sector is solved using the lookup table.

SI.5 Historical U.S. Investment and Profits

From the end of World War II through 1968, total U.S. gross investment was approximately 1.5 times profits (see Figure S.3(a)). It then rose to near 2.5 in the mid-1980s before dropping below 1.5 in 2009 after the global financial crisis. Between 1985 and 2009, investment relative to profits reached a peak of 2.6 in 2000 at the height of the dot-com bubble. In 2010, following the global financial crisis, investment dropped to 1.2 times profits before reaching approximately 1.5 in 2016. There are two anomalous time periods for investment and profits, seen via data not in the upper right portion Figure S.3(b). These two anomalous time periods are when U.S. private investment was less than profits during World War II and when corporate profits were negative during the depths of the Great Depression.

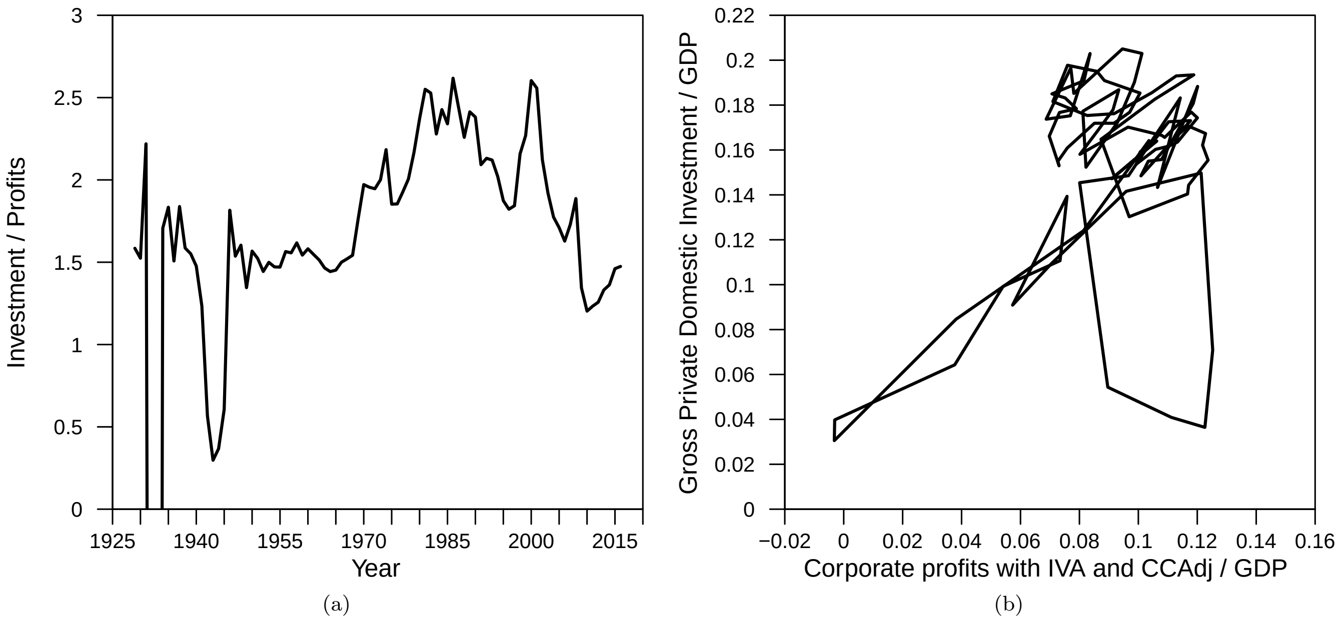

Figure S.3: Since World War II, U.S. gross private investment has varied from 125% to 260% of corporate profits, typically residing from 150% to 200% of profits. (a) Investment over profits versus time. (b) Investment versus profits as is used to inform the investment behavior of Equation 6. Data are from Bureau of Economic Analysis Tables 1.1.5 (GDP and gross investment) and 1.1.12 (corporate profits with inventory valuation adjustment, IVA and capital consumption adjustment, CCAdj).

SI.6 Constant Cost Markup simulations varying markup and investment function parameters κ_0 and κ_1

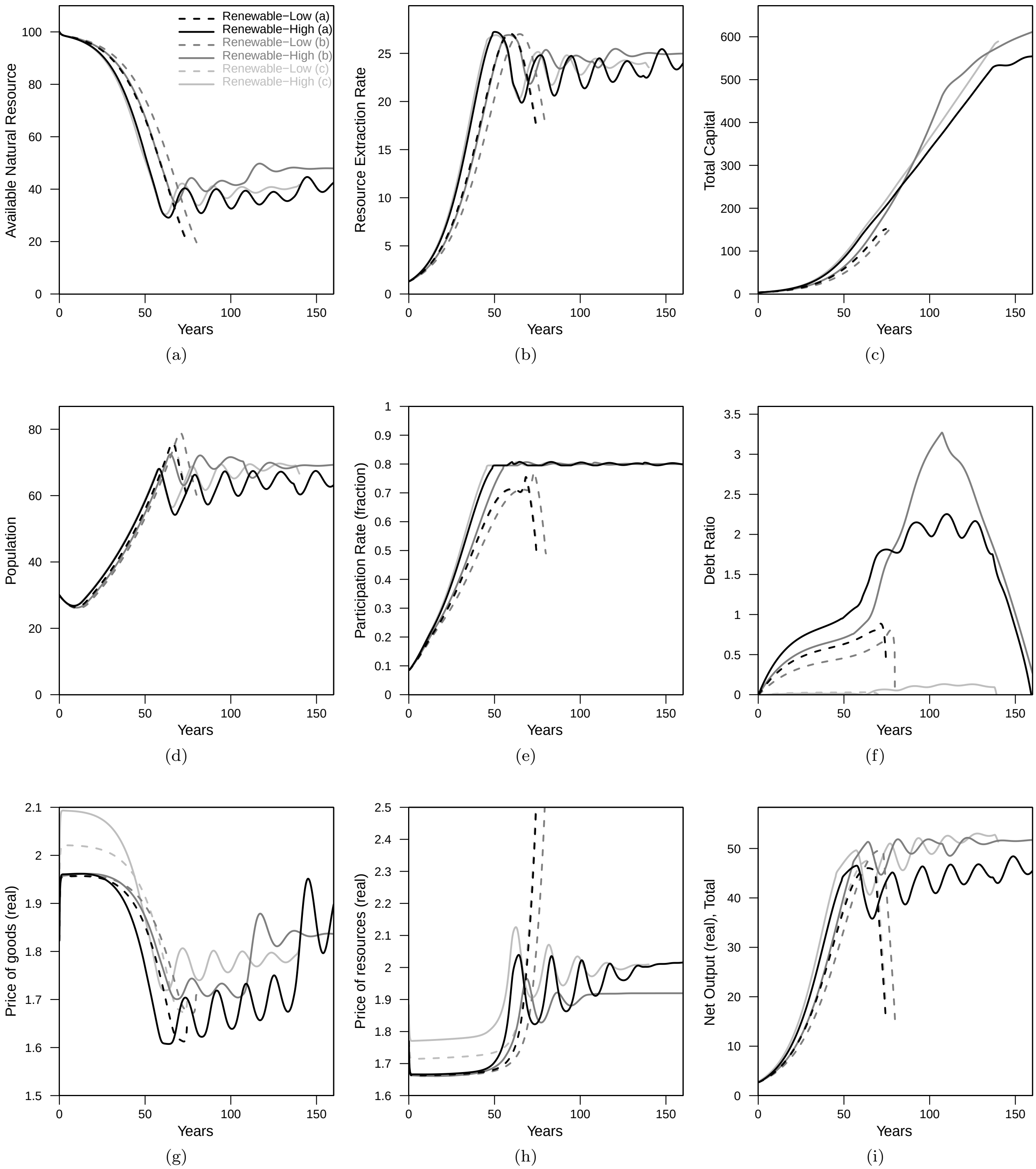

Figure S.4: Simulation results when setting prices via a constant markup. Renewable-Low scenarios are shown by dashed lines, and Renewable-High scenarios are shown by solid lines. Black lines hold $\kappa_0 = 1.0$ and $\mu_i = 0.07$ and use $\kappa_1 = 1.3$ (dashed) and $\kappa_1 = 1.5$ (solid). Dark gray lines hold $\kappa_1 = 1.0$ and $\mu_i = 0.07$ and use $\kappa_0 = 1.3$ (dashed) and $\kappa_0 = 1.5$ (solid). Light gray lines hold $\kappa_0 = 1.015$ and $\kappa_1 = 1.015$ and use $\mu_i = 0.09$ (dashed) and $\mu_i = 0.11$ (solid). (a) available resources (in the environment), (b) resource extraction rate, (c) total capital, (d) population, (e) participation rate, (f) debt ratio, (g) real price of goods, (h) real price of extracted resources, (i) total real net output, and (continued) ...

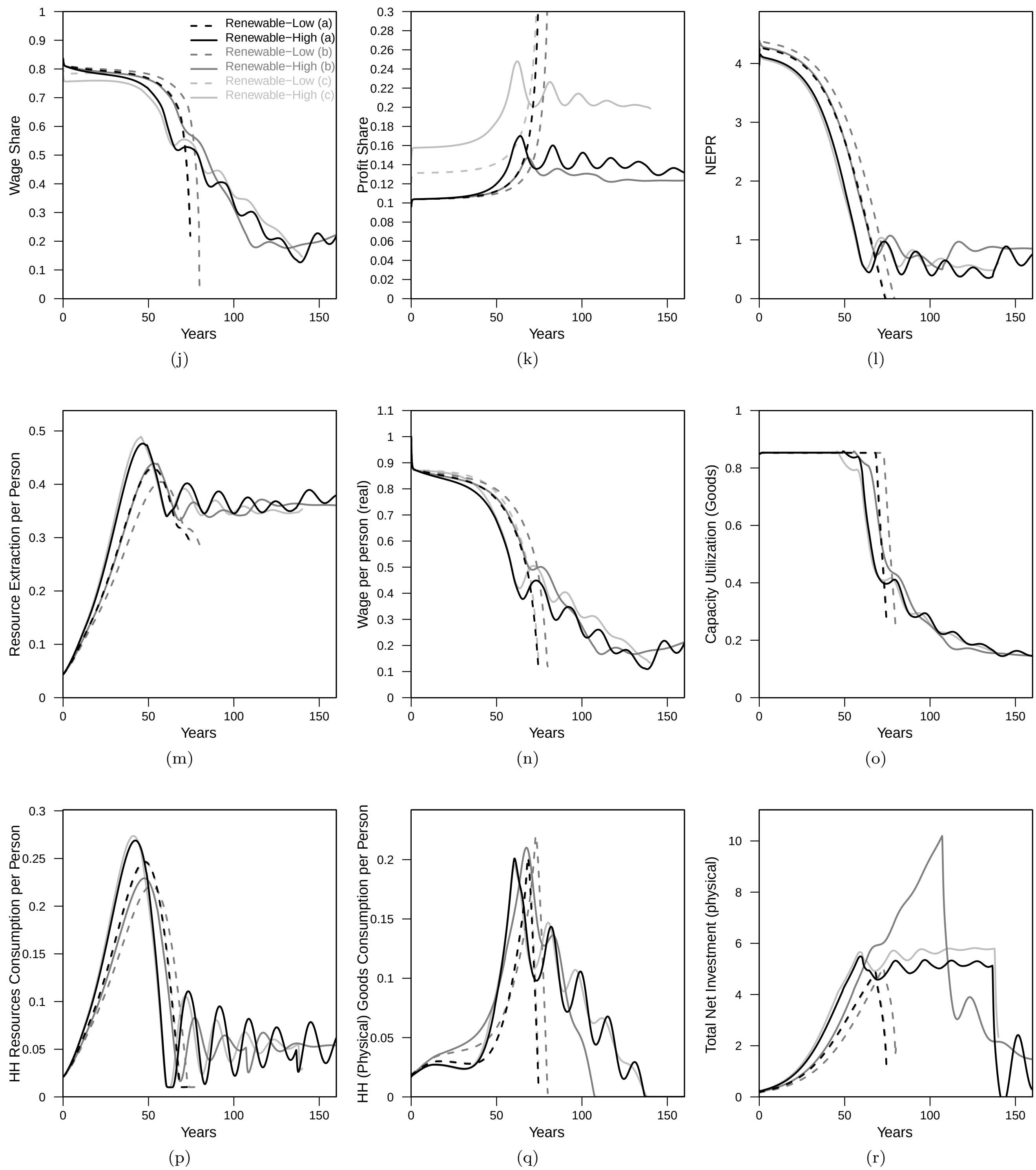

Figure S.4: (continued) ... (j) wage share, (k) profit share, (l) net external power ratio (NEPR), (m) resource extraction per person, (n) real wage per person, (o) capacity utilization of goods capital, (p) household consumption of (physical) resources per person, (q) household consumption of (physical) goods per person, (r) physical net investment, ...

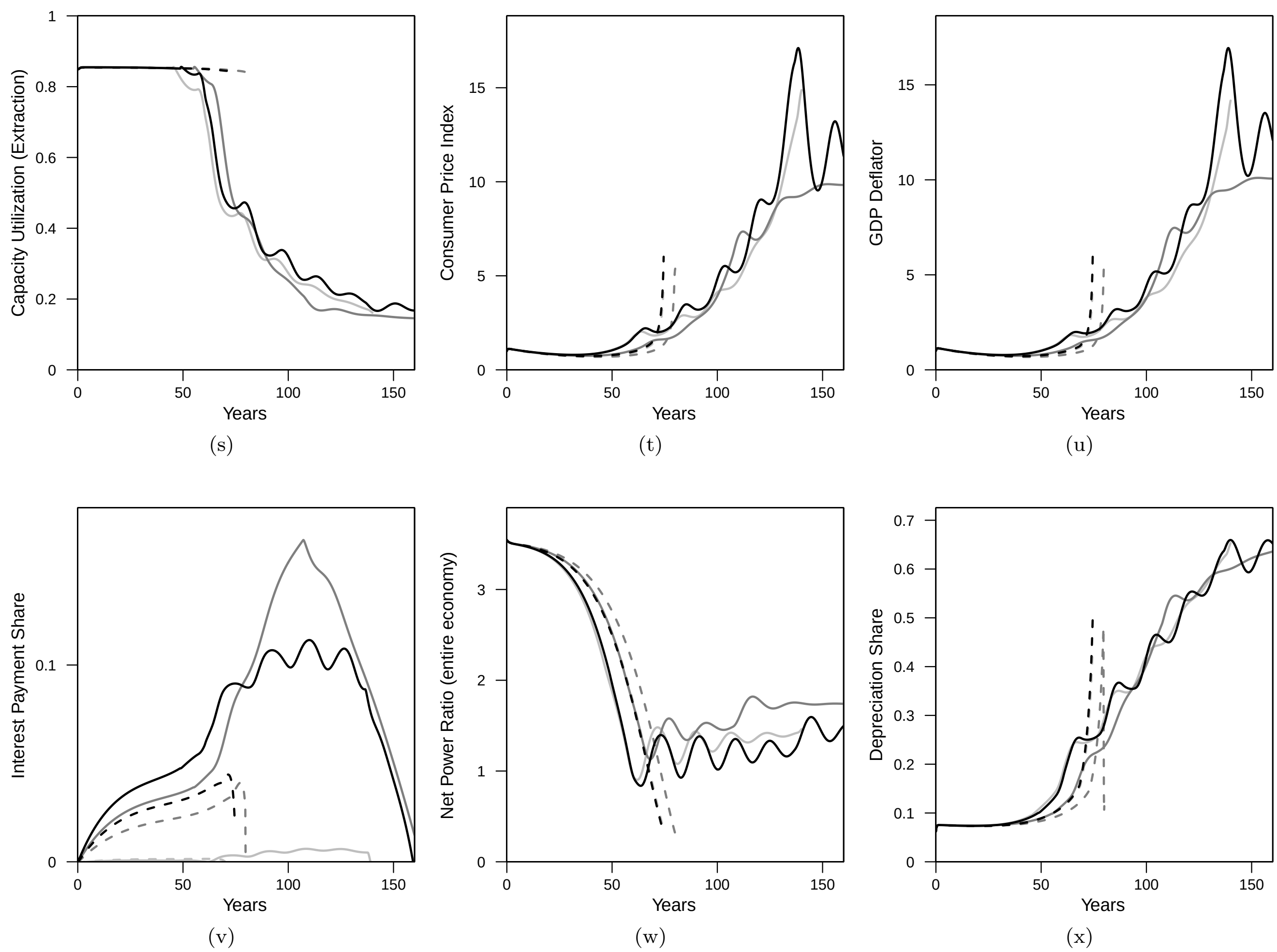

Figure S.4: (continued) ... (s) capacity utilization of extraction capital, (t) consumer price index, (u) GDP deflator, (v) share of value added allocated to interest payments, (w) economy-wide net power ratio, and (x) share of value added allocated to depreciation.

SI.7 Simulation Results: Variable cost markup

The results plotted in Figure S.5 assume the variable cost markup of Equation 19. As in the constant markup scenarios we vary the investment function parameters. Otherwise we use the same parameter values as in the constant cost markup simulations except for the extraction parameter δ_y. We set $\delta_y = 0.019$ such that the steady state amount of available resources is approximately $y \sim 40$, a value comparable to the long-term value of the Renewable-High constant markup scenarios that do not create rapid collapse. As in the constant markup Fossil scenario, for the variable markup Fossil scenario we increase λ_y by an order of magnitude ($\lambda_y = 1000$) and decrease δ_y by an order of magnitude ($\delta_y = 0.0019$). Of course the cost markup now varies, and thus is not similar to the constant markup scenarios.

The dynamics using the variable cost markup differ in that they are slower than those assuming a constant markup. The growth dynamics are slower because the variable markup declines relatively rapidly starting from about 0.09 and 0.04 for goods and extraction, respectively (Figure S.5(v) and (w)). The markup declines because the profit rate and profit share (Figure S.5(k)) for each sector declines exponentially from the start of the simulation.

Just as in the constant markup scenarios, the type and level of investment determines whether the system reaches a full collapse. The scenarios with $\kappa_0 > 1.0$ and $\kappa_1 = 1.0$ reach a full collapse as they run out of net resource extraction and reach NEPR=0 before debt ratios peak and decline to zero. This is a similar dynamic as the Renewable-Low scenarios. The scenarios with $\kappa_0 = 1.0$ and $\kappa_1 > 1.0$ avoid a full rapid collapse. Over the very long-term these scenarios continue to accumulate capital, thus continually increasing the depreciation share and decreasing wage share even after debt ratio becomes zero. This is the same long-term dynamic as the constant markup Renewable-High scenarios, albeit much slower paced.

Slower investment in the variable markup scenarios leads to higher peak population growth. For the scenarios that do not collapse, the steady state population is 10%-15% higher (Figure S.5(d)) as compared to the constant markup scenarios (Figure S.4(d)). The system reaches this stable population simultaneously upon reaching the peak and steady state per capita resources extraction rate (Figure S.5(m)), and maximum participation rate (Figure S.5(e)). Participation rate initially rises and declines during the first 100 years before rising to its maximum level as the population declines. Larger κ_0 or κ_1 slightly delays an initial peak in participation rate (for $T < 100$) but ultimately reaches maximum participation rate sooner. Interestingly, the system reaches peak debt ratio (Figure S.5(f)) just before reaching constant population and maximum participation rate.

The larger the investment via increased parameters κ_0 or κ_1, the earlier the peak in resources extraction, population, per capita goods consumption, and real net output. The peak debt ratio increases for $1 < \kappa_1 < 1.2$ but decreases after $\kappa_1 > 1.2$. Larger κ_1 also causes and earlier peak in debt ratio. Thus, "moderate" rates of investment lead to the highest peak debt ratio. With higher debt ratio comes higher share of value added to interest payments, and wage share and real wages mirror the interest share during the first half of the simulations.

Comparing the non-collapsing variable markup scenarios to the constant markup Renewable-High scenarios, real net output eventually reaches a similar, but slightly higher value at a lower amount of accumulated capital. With lower investment fewer resources are allocated to capital investment, and more is left for consumption. Further, the trends for the real price of goods and resources are similar, but more muted, than in the constant markup scenarios. The real price of goods declines until household consumption of physical goods reaches its peak. The real price of resources is relative steady before increasing as the population declines to its steady state value.

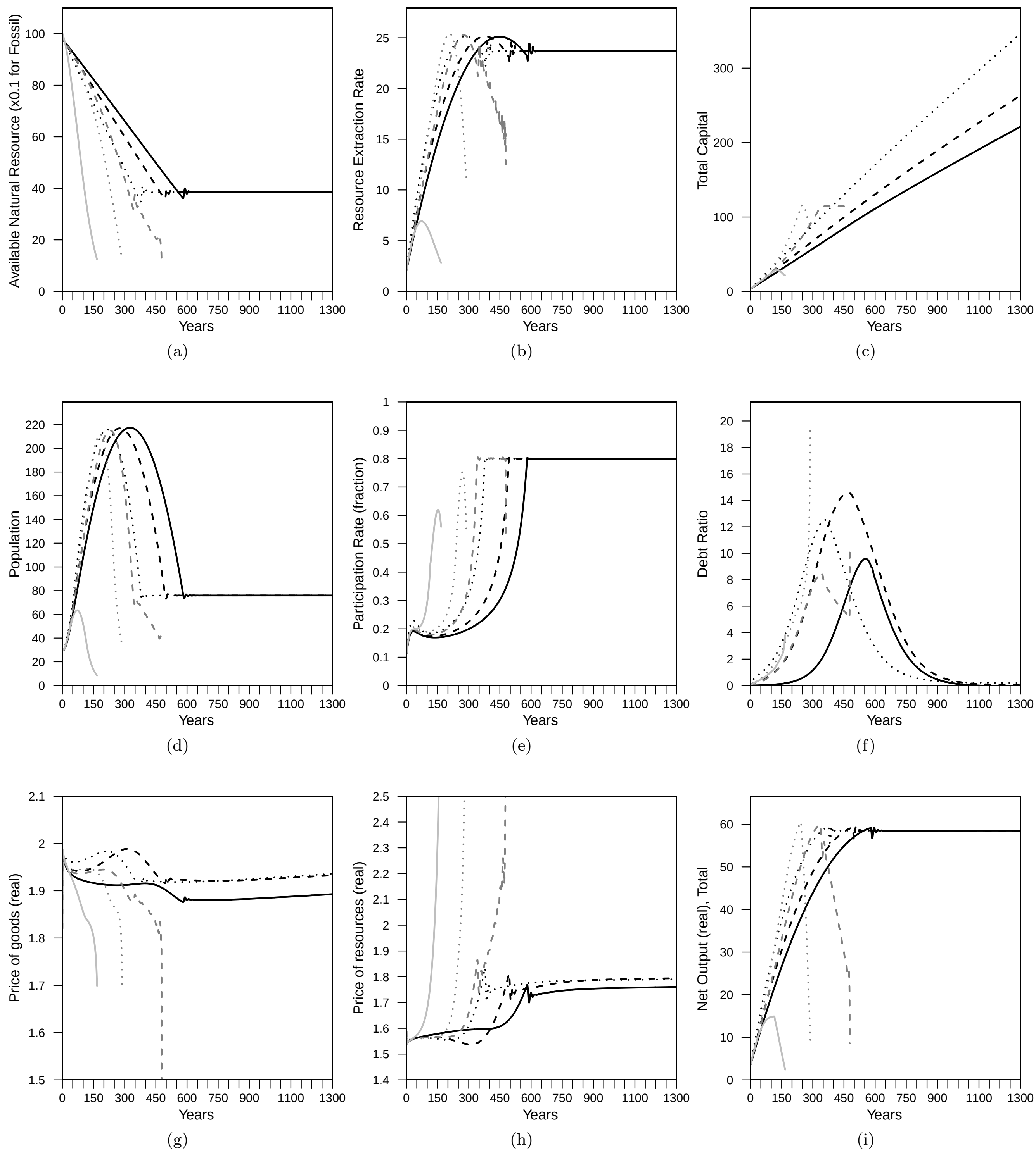

Figure S.5: Simulation results with prices set using the variable cost markup specific to each sector. Assuming a renewable resource ($\gamma = 0.01$) the investment function parameters of Equation 6 are set as $\kappa_0 = 1.0$ and κ_1 as 1.0 (black solid line), $\kappa_0 = 1.0$ and $\kappa_1 = 1.2$ and 1.6 (black dashed and dotted lines, respectively), $\kappa_1 = 1.0$ and $\kappa_0 = 1.1$ and 1.2 (dark gray dashed and dotted lines, respectively). Assuming a fossil resource ($\gamma = 0.00$) the investment parameters are $\kappa_0 = 1.0$ and $\kappa_1 = 1.2$ (light gray solid line). Simulation results as (a) available resources (in the enviroment), (b) resource extraction rate, (c) total capital, (d) population, (e) participation rate, (f) debt ratio, (g) real price of goods, (h) real price of extracted resources, (i) total real net output, (continued) ...

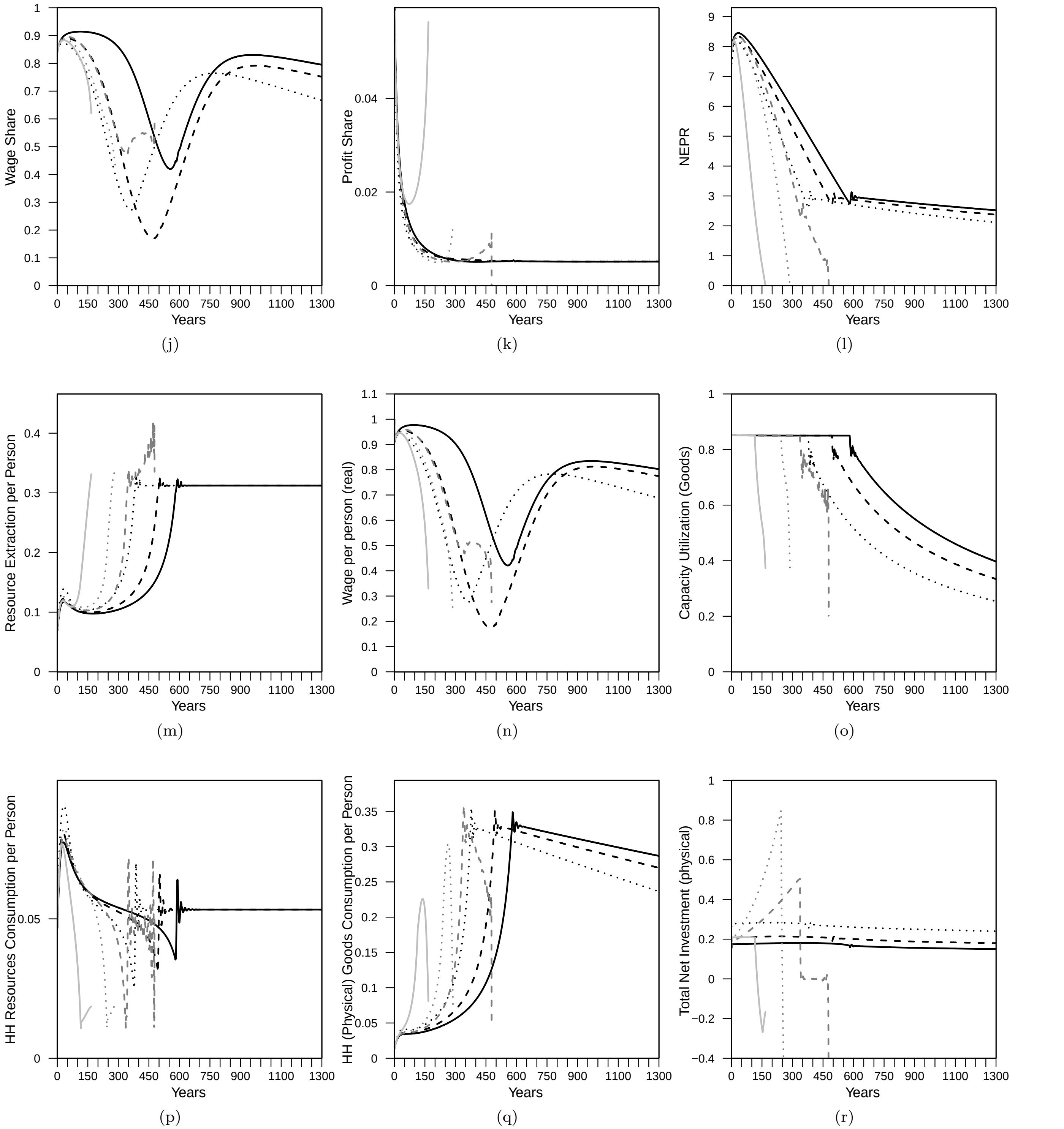

Figure S.5: (continued) ... (j) wage share, (k) profit share, (l) net external power ratio (NEPR), (m) net extraction per person, (n) real wage per person, (o) capacity utlization (goods sector), (p) household consumption of (physical) resurces per person, (q) household consumption of (physical) goods per person, (r) physical net investment, (continued) ....

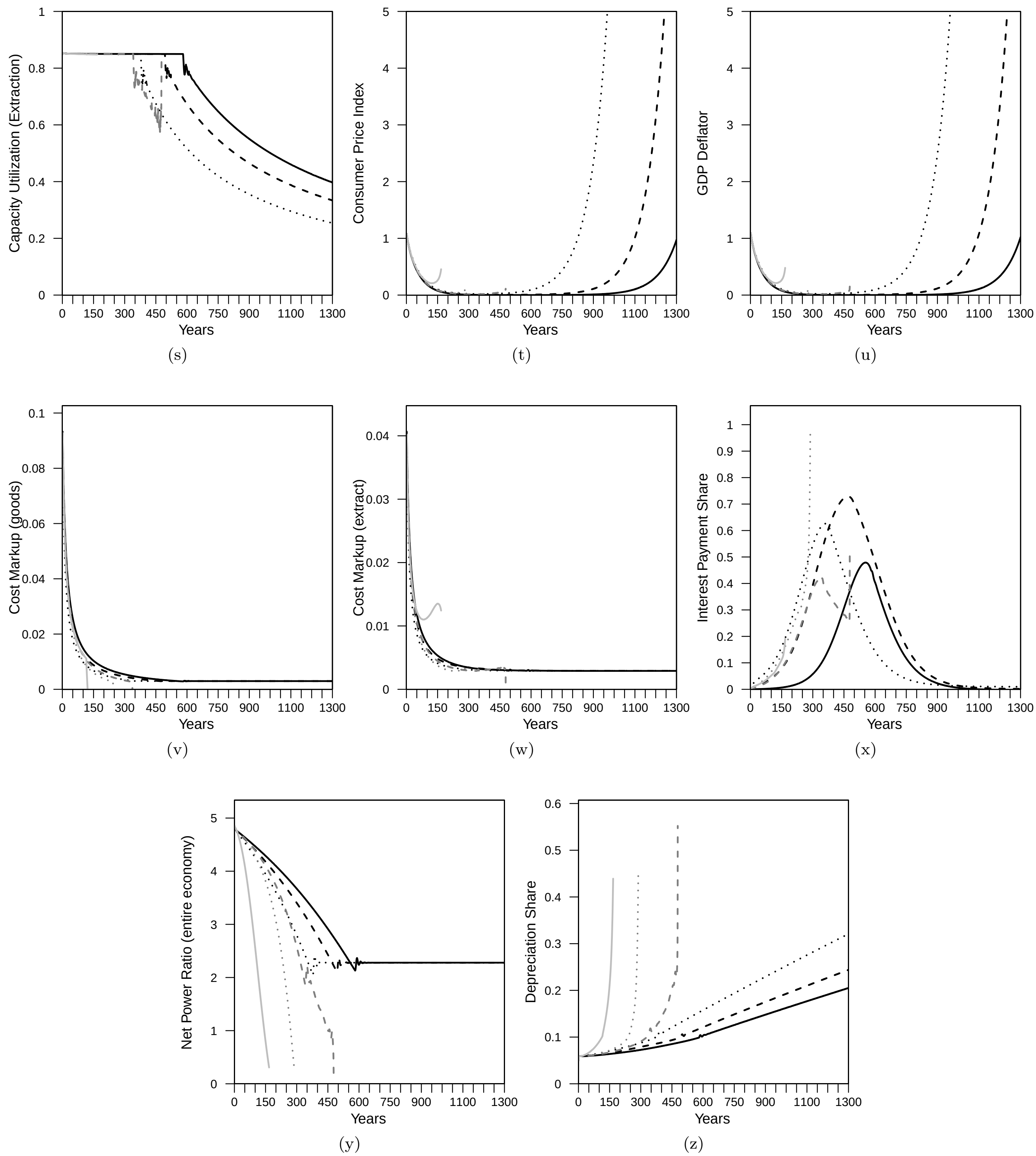

Figure S.5: (continued) ... (s) capacity utilization of extraction capital, (t) consumer price index, (u) GDP deflator, (v) cost markup for goods sector, (w) cost markup for extraction sector, (x) share of value added allocated to interest payments, (y) economy-wide net power ratio, and (z) share of value added allocated to depreciation.

SI.8 List of differential equations for model

Table S.3: The differential equations of the merged biophysical and economic model.

State Variable	Equation	Equation No.
I. Core equations		
Available resources	$\dot{y} = \gamma y(\lambda_y - y) - \delta_y y K_e CU_e$	3
Population	$\dot{N} = \beta_N N - \alpha_N \left(\frac{C_e}{P_e}\right) N$	32
Wealth	$\dot{w}_H = (IC_{ref,e} - IC_{percieved,e})\times$ $(C_e/P_e + a_{eg}X_g + a_{ee}X_e)$	25
Goods	$\dot{g} = (IC_{ref,g} - IC_{percieved,g})\times$ $((C_g + I_g + I_e)/P_g + a_{ge}X_e + a_{gg}X_g)$	26
Extraction capital	$\dot{K}_e = I_e/P_g - \delta K_e$	5
Goods capital	$\dot{K}_g = I_g/P_g - \delta K_g$	5
Debt for extraction sector	$\dot{D}_e = I_e - P_g \delta K_e - \Pi_e$	7
Debt for goods sector	$\dot{D}_g = I_g - P_g \delta K_g - \Pi_g$	7
Wage per capita	$\dot{w} = \phi(\lambda_N) \times w$	8
II. Lag equations for accessing 'past' states needed to solve core equations		
Capacity utilization for extraction sector	$\dot{CU}_e = \frac{CU_{e,indicated} - CU_e}{\tau_{CU,e}}$	of form of S.13
Capacity utilization for goods sector	$\dot{CU}_g = \frac{CU_{g,indicated} - CU_g}{\tau_{CU,g}}$	of form of S.13
Perceived inventory coverage for extraction sector	$\dot{IC}_{e,perceived} = \frac{IC_e - IC_{e,perceived}}{\tau_{IC,e}}$	of form of S.13
Perceived inventory coverage for goods sector	$\dot{IC}_{g,perceived} = \frac{IC_g - IC_{g,perceived}}{\tau_{IC,g}}$	of form of S.13
Price of extraction sector output	$\dot{P}_{e,lagged} = \frac{P_e - P_{e,lagged}}{\tau_{P,e}}$	of form of S.13
Price of goods sector output	$\dot{P}_{g,lagged} = \frac{P_g - P_{g,lagged}}{\tau_{P,g}}$	of form of S.13
Value added for extraction sector	$\dot{V}_{e,lagged} = \frac{V_e - V_{e,lagged}}{\tau_{V,e}}$	of form of S.13
Value added for goods sector	$\dot{V}_{g,lagged} = \frac{V_g - V_{g,lagged}}{\tau_{V,g}}$	of form of S.13
Net output from extraction sector	$\dot{Y}_{e,lagged} = \frac{Y_e - Y_{e,lagged}}{\tau_{Y,e}}$	of form of S.13
Net output from goods sector	$\dot{Y}_{g,lagged} = \frac{Y_g - Y_{g,lagged}}{\tau_{Y,g}}$	of form of S.13
Profit from extraction sector	$\dot{\Pi}_{e,lagged} = \frac{\Pi_e - \Pi_{e,lagged}}{\tau_{\Pi,e}}$	of form of S.13
Profit from goods sector	$\dot{\Pi}_{g,lagged} = \frac{\Pi_g - \Pi_{g,lagged}}{\tau_{\Pi,g}}$	of form of S.13
III. Lag equations (only for recording of variables of interest for post processing, all assumed using time delay of prices)		
Investment of each sector	$\dot{I}_i = \frac{I_i - I_{i,lagged}}{\tau_{P,i}}$	of form of S.13
Household consumption of each sector	$\dot{C}_i = \frac{C_i - C_{i,lagged}}{\tau_{P,i}}$	of form of S.13
Labor of each sector	$\dot{L}_i = \frac{L_i - L_{i,lagged}}{\tau_{P,i}}$	of form of S.13