Why climate sensitivity may not be so unpredictable

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[1] Different explanations have been proposed as to why the range of climate sensitivity predicted by GCMs has not lessened substantially in the last decades, and subsequently if it can be reduced. One such study (*Why is climate sensitivity so unpredictable?*) addressed these questions using rather simple theoretical considerations and reached the conclusion that reducing uncertainties on climate feedbacks and underlying climate processes will not yield a large reduction in the envelope of climate sensitivity. In this letter, we revisit the premises of this conclusion. We show that it results from a mathematical artifact caused by a peculiar definition of uncertainty used by these authors. Applying standard concepts and definitions of descriptive statistics to the exact same framework of analysis as Roe and Baker, we show that within this simple framework, reducing inter-model spread on feedbacks does in fact induce a reduction of uncertainty on climate sensitivity, almost proportionally. Therefore, following Roe and Baker assumptions, climate sensitivity is actually not so unpredictable. **Citation:** Hannart, A., J.-L. Dufresne, and P. Naveau (2009), Why climate sensitivity may not be so unpredictable, *Geophys. Res. Lett.*, *36*, L16707, doi:10.1029/2009GL039640.

1. Introduction

[2] Uncertainties in projections of future climate change described in the last Assessment Report of the IPCC [*Intergovernmental Panel on Climate Change*, 2007] are high, as illustrated by the broad range of climate sensitivity - defined as the global mean temperature increase for a doubling of CO_2 - simulated by general circulation models (GCMs). Attempts to explain this fact have focused mainly on uncertainties in our understanding of the individual physical feedback processes, difficulties to represent them faithfully in GCMs, nonlinearity of some processes and complex interactions among them giving rise to a chaotic behaviour of the climate system [*Randall et al.*, 2007]. A review of these explanations is given by *Bony et al.* [2006]. Nevertheless, in this letter, we leave aside these considerations to focus our interest solely on the explanation proposed by *Roe and Baker* [2007] (hereinafter referred to as RB07) which somewhat differ from the above-mentioned. This study uses the framework of feedback analysis, which was often used to describe the relationship between physical processes and climate sensitivity (see for instance *Lu and Cai* [2008], *Dufresne and Bony* [2008], and *Soden and Held* [2006]). This framework assumes a linear approximation of radiative feedbacks, resulting in a simple relationship between a global feedback gain f and climate sensitivity ΔT. In this classic setting, the main originality of RB07 approach consists in analyzing explicitly the way uncertainties on f, due to a limited understanding of their underlying physical processes, propagates into uncertainties on ΔT: assuming f is a random variable with mean $\overline{f}$ and standard deviation σ_f, RB07 uses this simple probabilistic model to highlight several fundamental properties of uncertainty propagation from feedbacks to climate sensitivity. The most prominent conclusion of this analysis is that reducing uncertainties on f does not yield a large reduction in the uncertainty of ΔT, and thus that improvements in the understanding of physical processes will not yield large reductions in the envelope of future climate projections. This conclusion, if true, would clearly have crucial implications for climate research and policy.

[3] In section 2, we revisit the premises of RB07 conclusion. We highlight that it is the result of a peculiar way of defining uncertainty. Moreover, we show in section 5 that this conclusion is a mathematical artifact with no connection whatsoever to climate. Since the basic question of the definition of uncertainty appears to be at stake, section 3 briefly recalls widely used definitions and elementary results on uncertainty and its propagation as can be found in Descriptive Statistics textbooks. In section 4, we apply these standard definitions to the exact same framework of analysis as RB07. We show that within this framework, reducing inter-model spread on feedbacks does in fact induce a reduction of uncertainty on climate sensitivity, almost proportionally. Finally, section 6 concludes.

2. Overview of RB07 Approach

[4] RB07 uses the feedback analysis framework. Denoting ΔT_0 the Planck temperature response to the radiative perturbation and f the feedback gain (RB07 refers to it as feedback factor), they obtain:

$$\Delta T = \frac{\Delta T_0}{1 - f} \tag{1}$$

RB07 then assumes uncertainty on Planck response to be negligible so that the entire spread on ΔT results from the uncertainty on the global feedback gain f. To model this uncertainty, RB07 assumes that f follows a Gaussian distribution with mean $\overline{f}$, standard deviation σ_f and implicit truncation for $f > 1$ (Appendix A1). Then, they derive an exact expression of the distribution of ΔT. This simple probabilistic climatic model is used by RB07 to analyze the way uncertainties on f, due to a limited understanding of underlying physical processes, propagates into uncertainties on ΔT. Their analysis highlights two fundamental properties:

[5] 1. Amplification: The term in $\frac{1}{1-f}$ in equation (1) amplifies uncertainty on feedbacks, all the more intensely as

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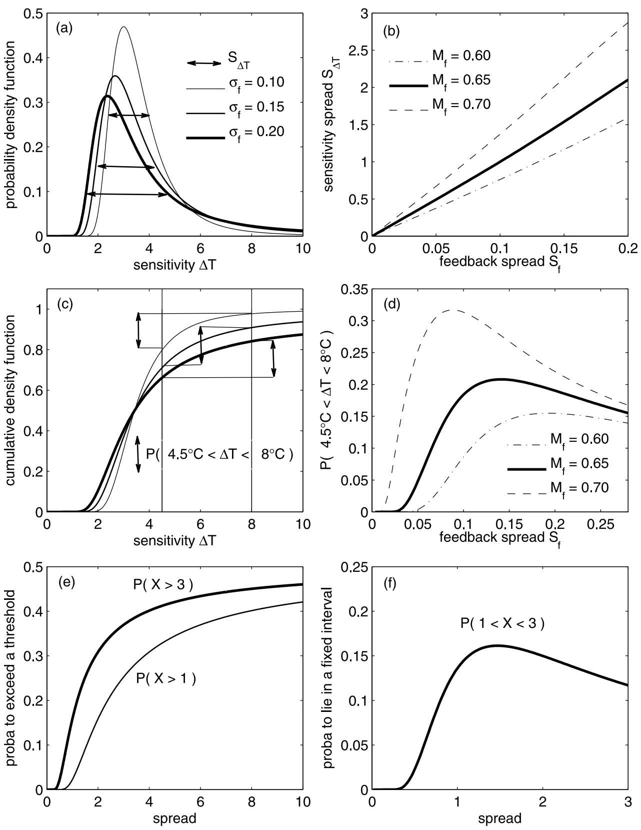

Figure 1. Feedback gain f is truncated Gaussian $\mathcal{N}$ (M_f, σ_f) as by RB07. X is centered Gaussian $\mathcal{N}$ (0, S_X). (a) Pdf of ΔT with M_f = 0.65 and σ_f = 0.20, 0.15, 0.10. Arrows represent the decreasing sensitivity spread $S_{\Delta T}$ obtained for decreasing values of σ_f. (b) Climate sensitivity spread $S_{\Delta T}$ as a function of feedback spread S_f, for M_f = 0.60, 0.65, 0.70. Feedback spread S_f is measured by standard deviation ($\simeq\sigma_f$) but climate sensitivity spread $S_{\Delta T}$ is measured by IQR (see Appendix A1 for explanation). (c) Cdf of ΔT. Arrows represent the stable probability $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ obtained for decreasing values of σ_f = 0.20, 0.15, 0.10. (d) Probability $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ as a function of feedback spread S_f measured with standard deviation. (e) Probability for X to exceed respectively 1 and 3, as functions of spread S_X. (f) Probability for X to fall within interval [1, 3] as a function of spread S_X.

$\overline{f}$ is close to (though lower than) one. Small uncertainties on feedbacks are thus converted in large uncertainties on the rise of temperature.

[6] 2. Insensitivity: Quoting RB07, *"reducing uncertainty on f has little effect in reducing uncertainty on* ΔT*",* also stated as *"the breadth of the distribution of* ΔT *is relatively insensitive to decreases in* σ_f*."*

[7] We fully subscribe to the first property and elaborate further on it in section 4. However, we are puzzled by the second property, that is, the claimed insensitivity of uncertainty on ΔT to uncertainty on feedbacks. The reason why one may find this second assertion puzzling, is that it intuitively seems to contradict the first. Indeed, if small uncertainties on f are amplified into large uncertainties on ΔT, it suggests that a strong dependency exists between both uncertainties, rather than no or little dependency. We therefore dig into the details of RB07 argumentation regarding this assertion. To get to that conclusion, RB07 actually focus on the probability of large temperature increase defined as $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$. They study graphically how this probability fluctuates with the level of uncertainty on feedbacks, by plotting the cumulative distribution of ΔT for several values of σ_f. Doing this graphical analysis, they observe that $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ is insensitive to σ_f. This observation is easily verifiable: we replicated RB07 cumulative distribution chart in Figure 1c, and we computed several values of $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ for $\overline{f}$ = 0.65 and σ_f ranging from 0.10 to 0.20, finding it to fluctuate between 0.18 and 0.20. Therefore, in agreement with RB07, it is fair to say that the probability of large temperature increase is quite insensitive to σ_f in this domain. However, concluding from this observation that *"the breadth of the distribution of* ΔT *is relatively insensitive to decreases in* σ_f*"* implicitly assumes two very different definitions of uncertainty: while on the side of feedback the uncertainty is measured by standard deviation σ_f, on the side of sensitivity the probability $\mathbb{P}(\Delta T \in [4.5°C, 8°C])$ is used as a metric of uncertainty. As will be developed in section 3, standard deviation is a standard, consensual uncertainty metric but the probability to lie in a fixed interval is not. While under this peculiar double definition of uncertainty RB07 conclusion holds, it is fair to ask whether it would still hold with a different uncertainty metric for ΔT; second, whether the probability to lie in a fixed interval can be considered an acceptable measure of distribution breadth; and third, what are the implications of using such an asymmetric definition of uncertainty. The following sections attempt to answer these questions.

3. Standard Measurement and Propagation of Distribution Spread

[8] To investigate the first question, which relates to the basic issue of defining uncertainty, we briefly recall a few standard definitions and concepts, as they can be found almost identically in most Descriptive Statistics textbooks. For details, the reader can refer for instance to *Barlow* [1989], *Reinard* [2006], and *James and Eadie* [2006] to mention but a few such textbooks.

[9] Descriptive Statistics provide metrics summarizing a sample of observations and, in probabilistic terms, the probability density function (pdf) underlying them. Technically, the correspondence between both is simply that a sample summary is an *estimator* (a function of the data) which estimates a pdf summary *estimand* (a function of its parameters). Metrics are usually grouped under three categories: location, scale and shape parameters. The so-called location parameters are meant to identify the center of a distribution. The most common location measures are mean, mode and median. The so-called scale parameters, also referred to as dispersion, variability, variation, scatter or spread measures, describe how far from the above-defined center possible values covered by the distribution tend to be. This second group of metrics is the one we are interested in for our discussion, as it is concerned with the measurement of distribution spread. The most common measures are standard deviation, interquartile range (IQR), range or

median absolute deviation (MAD), more rarely full width at half maximum (FWHM); they are dimensionally homogeneous to the variable. Variance and coefficient of dispersion are commonly used inhomogeneous spread measures. The cited references give complete expressions, properties, strengths and limitations of these measures. We underline a property of interest to our discussion: homogeneous spread measures are invariant in translation and linear in scale. In other words, denoting $S_{\mathbf{X}}$ any measure of spread, X a random variable and $Y = aX + b$ then:

$$S_Y = |a| \,.\, S_X \tag{2}$$

Further, in the general case of a dependency of type $Y = \phi(X)$:

$$S_Y \simeq |\phi'(M_X)| \,.\, S_X \tag{3}$$

where ϕ' represents the first derivative of ϕ and M is a location parameter. This linear approximation is commonly used to combine errors on measurements, though generally in its multivariate formulation, and is thus sometimes referred to as the error propagation framework. It may also be used to study the way uncertainty on some input variable(s) propagates into uncertainty on an output obtained from a determinist function, as in section 4.

4. Standard Propagation of Feedback Uncertainty in RB07 Model

[10] We now analyze the dependency between uncertainty on feedbacks and uncertainty on climate sensitivity in the RB07 model. Denoting $S_{\Delta T}$ a measure of climate sensitivity spread, S_f a measure of feedback spread and M_f a measure of feedback location, the uncertainty propagation formula (3) is applied to equation (1) leading to:

$$S_{\Delta T} \simeq \frac{\Delta T_0}{\left(1 - M_f\right)^2} \,.\, S_f \tag{4}$$

Equation (4) holds for any choice of pdf for feedback gain f and thus applies more generally than in the particular case of a truncated Gaussian pdf chosen by RB07. It provides a simple relationship between $S_{\Delta T}$, S_f and M_f which translates into the following two properties:

[11] 1. Amplification: In agreement with RB07 first above recalled result, for a fixed level of feedback uncertainty S_f, the level of sensitivity uncertainty $S_{\Delta T}$ is amplified when feedback M_f approaches one. Since estimates of feedback parameters in CMIP3/AR4 models [*Soden and Held*, 2006; *Randall et al.*, 2007] suggest M_f is close enough to one ($M_f \simeq 0.65$), it seems that *"the climate system is operating in a regime in which small uncertainties in feedbacks are amplified in the resulting climate sensitivity uncertainty"*, to quote RB07.

[12] 2. Proportionality: In disagreement with RB07 second above recalled result, for a fixed level of average feedback M_f, the level of climate sensitivity uncertainty $S_{\Delta T}$ is proportional to the level of feedback uncertainty S_f. This proportionality between uncertainties is intuitive. Indeed, when $S_f = 0$, feedbacks are determinists and ΔT also is, considering no other source of uncertainty in the climate system, hence $S_{\Delta T} = 0$. As values of f get increasingly scattered, resulting values of climate sensitivity also get more scattered proportionally (Figures 1a and 1b).

[13] This proportionality has general validity in the sense that it holds for any homogeneous spread measure and for any distribution of f. However, it is an approximation for small values of S_f: we therefore analyzed how this linear dependency is affected when S_f increases. In this purpose, we exhibit more precise results on uncertainty propagation in the RB07 model. First, when spread is measured by IQR, an exact relationship holds for any value of S_f and any distribution of f (Appendix A2):

$$S_{\Delta T} = \frac{\Delta T_0}{\left(1 - M_f\right)^2}\, S_f \,.\left\{1 - \frac{w_f}{1 - M_f}\, S_f - \frac{1 - w_f^2}{4\left(1 - M_f\right)^2}\, S_f^2\right\}^{-1} \tag{5}$$

where w_f measures the asymetry of f distribution. Hence, when $S \equiv$ IQR, the dependency is overlinear when f has a symetric or right skewed pdf. Otherwise, the dependency is sublinear for small values of S_f but becomes overlinear when S_f is large enough. Second, when spread is measured by standard deviation, a second order Taylor expansion of equation (1) leads to a more accurate approximation (Appendix A3):

$$S_{\Delta T} \simeq \frac{\Delta T_0}{\left(1 - M_f\right)^2}\, S_f \,.\left\{1 + \frac{2w_f}{1 - M_f}\, S_f + \frac{k_f - 1}{\left(1 - M_f\right)^2}\, S_f^2\right\}^{\frac{1}{2}} \tag{6}$$

Again, overlinearity prevails when $w_f \geq 0$ or S_f large enough. Third, when S is standard deviation and f distribution is log-normal, an exact formula holds for any S_f:

$$S_{\Delta T} = \frac{\Delta T_0}{\left(1 - M_f\right)^2} \,.\, S_f \,.\left\{1 + \left[\frac{S_f}{1 - M_f}\right]^2\right\} \tag{7}$$

and is again overlinear. Finally, overlinear relationships can also be derived when the distribution of f is assumed to be gamma or beta (Appendix A4).

[14] To summarize the above discussion, its main outcome is rather intuitive and has actually nothing to do with climate: if the spread of feedback gain values decreases, the resulting spread of climate sensitivity values also decreases. Second, the dependency is linear for small feedback spreads and tends to get overlinear for larger values. Last, the proportionality coefficient sharply increases as feedback intensifies.

5. Properties of the Probability to Lie in a Fixed Interval

[15] We now focus on whether the probability to lie in a fixed interval can be considered an acceptable measure of distribution breadth, as implicitly done by RB07 to reach their main conclusion. We approach this question very generally: let X be a continuous random variable with location M_X and spread S_X and let $[a, b]$ be a fixed interval

near but above the center ($M_X < a < b$). When $S_X \to 0$ the variable becomes determinist ($X = M_X$) hence $\mathbb{P}(X \in [a, b])$ equals to zero. When $S_X \to +\infty$ the distribution covers such a wide range of values that $\mathbb{P}(X \in [a, b])$, the relative weight of interval $[a, b]$, decreases to zero (Appendix A5). Hence the dependency between $\mathbb{P}(X \in [a, b])$ and S_X is characterized by a non monotonous function that increases, flattens and then decreases to zero (Figure 1f). In light of this non monotonous dependency, it is difficult to hold $\mathbb{P}(X \in [a, b])$ as a valid measure for the width of X distribution. Further, the observed insensitivity of $\mathbb{P}(\Delta T \in [4.5°\mathrm{C}, 8°\mathrm{C}])$ to feedback spread S_f, which lead authors to their conclusion, happens to proceed directly from the above described dependency: this flattening of the dependency is a pure mathematical artifact which systematically manifests under these definitions, and has nothing to do with climate.

[16] Finally, if one still wants to stick to this peculiar, asymetric definition of uncertainty, it has to be noted that in the RB07 model, even though the dependency is flat in the domain $S_f \in [0.1, 0.2]$, the dependency is strong for $S_f < 0.1$ when $M_f \approx 0.65$ and subsequently leads to a steep decrease of $\mathbb{P}(\Delta T \in [4.5°\mathrm{C}, 8°\mathrm{C}])$ to zero (Figure 1d). In fact, since feedback current estimates suggest $S_f \simeq 0.09$ and $M_f \approx 0.65$ [*Soden and Held*, 2006; *Randall et al.*, 2007], the domain of strong dependency may actually already be reached to date.

6. Conclusion

[17] Developments in section 5 suggest that, while the probability $\mathbb{P}(\Delta T \in [4.5°\mathrm{C}, 8°\mathrm{C}])$ may be of interest practically, this metric is irrelevant to describe *"the breadth of the distribution of climate sensitivity"* which was RB07 explicit intent. To address this question, any measure of distribution spread chosen amongst those classically used in Descriptive Statistics and recalled in section 3, appear to us more appropriate. With such measures of spread, we showed in section 4 that in RB07 framework, when the spread of feedback parameter S_f decreases, the resulting spread of climate sensitivity $S_{\Delta T}$ values also decreases. Further, we also highlighted that in this framework, the decrease is approximately linear for S_f small and tends to be steeper for larger values of S_f.

[18] Other than the definition issue pointed here, the relevance of RB07 simplified model may also be discussed but this was beyond the scope of this letter. In any case, if one holds this model to be accurate, a decrease of the spread on feedback will lead to a decrease of the uncertainty on climate sensitivity and a narrowing of the envelope of future climate projections. If enough studies are undertaken to better understand involved physical processes, neither are doomed to remain at their current level.

Appendix A

A1. Implications of the Truncation

[19] Since the linear feedback model of RB07 implicitly assumes $f \leq 1$, the Gaussian distribution $\mathcal{N}(\overline{f}, \sigma_f)$ proposed by RB07 is implicitly truncated for $f > 1$. This truncation has several implications. First, σ_f (resp. $\overline{f}$) does not exactly match standard deviation (resp. mean) of the truncated distribution. For instance, when $(\overline{f}, \sigma_f) = (0.75, 0.25)$ the mean of f equals 0.67 and its standard deviation equals 0.18. Second, it introduces some negative skewness in the distribution of f (-0.39 in the same example). Finally, since the truncated Gaussian pdf is finite and non zero in $f = 1$, the obtained pdf of climate sensitivity behave as a Pareto distribution in $\mathcal{O}(\Delta T^{-2})$ for high values and does not have a finite mean nor a finite variance. Hence, the truncated Gaussian model of RB07 forbids the use of standard deviation as a measure of climate sensitivity spread, which explains the use of IQR in Figure 1. For the purpose of RB07 which is to study climate sensitivity spread, assuming a parametric distribution of f - such as log-normal, gamma or beta - which leads to finite mean and deviation for sensitivity and exact mathematical expressions of the dependency between the deviation of ΔT and the deviation of f would be in our view more convenient. However, results presented in section 4 are general and also hold for this truncated Gaussian. Therefore, RB07 truncated Gaussian is in our view inconvenient but it does not affect the main features of uncertainty propagation described in this letter.

A2. Exact Uncertainty Propagation Equation for IQR

[20] If X is a continuous random variable X, we denote X_α its α-quantile, $S_X = X_{0.75} - X_{0.25}$ its interquartile range, $M_X = X_{0.50}$ its median and $w_X = \frac{X_{0.75}+X_{0.25}-2X_{0.50}}{X_{0.75}-X_{0.25}}$ the so-called quartile skewness coefficient. We thus have $X_{0.75} = M_X + \frac{1}{2} S_X(1 + w_X)$ and $X_{0.25} = M_X - \frac{1}{2} S_X(1 - w_X)$. Since when Φ is a diffeomorphism, we also have $[\Phi(X)]_\alpha = \Phi(X_\alpha)$, hence from equation (1):

$$
\begin{aligned}
S_{\Delta T} &= \Delta T_{0.75} - \Delta T_{0.25} = \frac{\Delta T_0}{(1-f_{0.75})} - \frac{\Delta T_0}{(1-f_{0.25})} \\
&= \frac{\Delta T_0}{(1-f_{0.75})(1-f_{0.25})}\, S_f \\
&= \frac{\Delta T_0}{\left(1-M_f\right)^2}\, S_f \,.\left\{1 - \frac{w_f}{1-M_f}\, S_f - \frac{1-w_f^2}{4\left(1-M_f\right)^2}\, S_f^2\right\}^{-1}
\end{aligned}
$$

A3. Second Order Term in Uncertainty Propagation Equation

[21] A second order term Taylor expansion of ϕ about M_X is $Y \simeq \phi(M_X) + \phi'(M_X)\,(X - M_X) + \frac{1}{2}\phi''(M_X)\,(X - M_X)^2$. When the chosen spread measure S is standard deviation, calculations can be performed explicitly:

$$
\begin{aligned}
S_Y \simeq |\phi'(M_X)|\,.\,S_X\,.\Bigg\{1 &+ \left[\frac{\phi''(M_X)}{\phi'(M_X)} w_X\right] S_X \\
&+ \left[\frac{\phi''(M_X)^2}{4\phi'(M_X)^2}(k_X - 1)\right] S_X^2\Bigg\}^{\frac{1}{2}}
\end{aligned}
\tag{A1}
$$

Applying equation (A1) to model (1), equation (6) follows.

A4. Exact Uncertainty Propagation Equations for Standard Deviation

[22] Since the domain of value of f in the RB07 model is $]-\infty, 1]$, we assume single tailed distributions defined

on this support to avoid a truncation and make mathematical developments more convenient. For several usual distributions, the relationship between $S_{\Delta T}$ and S_f can be explicated. Assuming a log-normal distribution with pdf $\frac{1}{(1-f)\sigma\sqrt{2\pi}}$ exp $[-\frac{(\ln(1-f)-\mu)^2}{2\sigma^2}]$, mean $M_f = 1 - e^{\mu+\frac{\sigma^2}{2}}$ and variance $S_f^2 = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)$ we obtain $S_{\Delta T}^2 = \Delta T_0^2.\ e^{-2\mu+\sigma^2}(e^{\sigma^2} - 1)$. Recombining:

$$S_{\Delta T} = \frac{\Delta T_0}{\left(1 - M_f\right)^2}\ .S_f\ .\left\{1 + \left[\frac{S_f}{1 - M_f}\right]^2\right\} \tag{A2}$$

Assuming a gamma distribution with pdf $(1 - f)^{k-1} \frac{\exp\left(-(1-f)/\theta\right)}{\Gamma(k)\theta^k}$, mean $M_f = 1 - k\theta$ and variance $S_f^2 = \theta^2 k$, we obtain $S_{\Delta T}^2 = \Delta T_0^2.\ [\theta^2(k - 1)(k - 2)]^{-1}$. Recombining:

$$S_{\Delta T} = \frac{\Delta T_0}{\left(1 - M_f\right)^2}\ .S_f\ .\left\{1 - \left[\frac{S_f}{1 - M_f}\right]^2\right\}^{-1} .\left\{1 + \left[\frac{S_f}{1 - M_f}\right]^2\right\}^{-\frac{1}{2}} \tag{A3}$$

Assuming a beta distribution with pdf $\frac{\Gamma(2k)}{\theta\Gamma(k)^2}\ (1 - \frac{1-f}{\theta})^{k-1}$ $(\frac{1-f}{\theta})^{k-1}$ on $[1 - \theta, 1]$, mean $M_f = 1 - \frac{\theta}{2}$ and variance $S_f^2 = \theta^2\ [8k\ [8k + 4]^{-1}$, we obtain $S_{\Delta T}^2 = \Delta T_0^2.\ [k(2k - 1)].\ [\theta^2\ (k - 1)^2(k - 2)]^{-1}$. Recombining:

$$S_{\Delta T} = \frac{\Delta T_0}{\left(1 - M_f\right)^2}\ .S_f\ .\left\{1 - \left[\frac{S_f}{1 - M_f}\right]^2\right\}^{\frac{1}{2}}\left\{1 - 2\left[\frac{S_f}{1 - M_f}\right]^2\right\}^{\frac{1}{2}} \cdot\left\{1 - 3\left[\frac{S_f}{1 - M_f}\right]^2\right\}^{-1}\left\{1 - 5\left[\frac{S_f}{1 - M_f}\right]^2\right\}^{-\frac{1}{2}} \tag{A4}$$

A5. Dependency Between Spread and Probability Weight of an Interval

[23] Assume U is a random real variable with pdf $p(u)$, cdf $P(u)$, center M and spread one. For $S > 0$, we introduce $X = S.(U - M) + M$ which has pdf $\frac{1}{S}\, p(\frac{x-M}{S} + M)$, cdf $P(\frac{x-M}{S} + M)$, center M and spread S. To analyze the dependency between S and the probability to lie in $[a, b]$ we study $F(S) = \mathbb{P}(X \in [a, b])$. F can be expressed by $F(S) = P(\frac{b-M}{S} + M) - P(\frac{a-M}{S} + M)$; hence $F(0) = P(-\infty) - P(-\infty) = 0$ and $F(+\infty) = P(M) - P(M) = 0$.

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